Strange equlibria of elastic rods: elastic arm scale, torsional gun, and the dripping of the Euler's elastica

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The problem of an elastic rod deforming in a plane, namely the so-called ‘planar elastica’, has a long history, rooting to Jacob Bernoulli (1654-1705), Daniel Bernoulli (1700-1782), Leonhard Euler (1707-1783), and Pieter van Musschenbroek (1692-1761), but is still actual and rich of applications, sometimes unexpected. The elastica has attracted a great interest in the past and has involved contributions from first-class scientists, including Kirchhoff, Love, and Born. The research on the elastica marked the initiation of the calculus of variations and promoted the development of the theory of elliptic functions. Nowadays the elastica represents a useful introduction to the theory of nonlinear bifurcation and stability, but is also an important tool in the field of soft robotics and in the design of compliant mechanisms. Moreover, the elastica can be effectively used to explain snake or fish locomotion and to design snake-like robots.

In this talk, the theory of a planar, nonlinear elastic rod developed by Euler is used to present new phenomena in which nonlinearities (related to the fact that equilibrium of the rod is reached at large displacements) play a fundamental role.

The first of these phenomena is buckling in tension: while we all know that a compressed elastic rod at a certain load bends, or buckles (Fig. 1, left), can we imagine that the same phenomenon may occur if the load is tensile? This has indeed been shown to occur, being related to the presence of a particular junction, a so-called ‘slider’ (Fig. 1, center), in the middle of the rod [1], or to the presence of a curved constraint [2]. Note that the shape of the rod buckled in tension is similar to the shape of a water meniscus in air (Fig. 1, right).

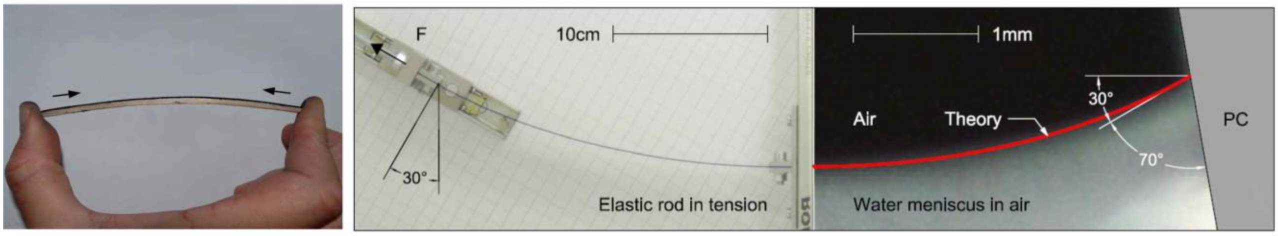

Fig. 1 Left: buckling of an elastic rod subject to compression; Center: buckling in tension; Right: analogy with a water meniscus in air

Flutter instability is an example of a Hopf bifurcation, related to the fact that a structure (the elastic rod) is subject to a nonconservative load. An instability of this type produced the collapse of the Tacoma narrow bridge. In that case the nonconservative force was an aeroelastic interaction. Hopf instabilities are also predicted to occur in granular media [3] and are believed to be related to intergranular friction. But how friction, a mere dissipation, can induce flutter, which is an oscillating motion of blowing-up amplitude? The answer to this question was given in an indisputable way through the realization of a Ziegler pendulum, in which the follower force is induced by dry friction [4], see Fig. 2.

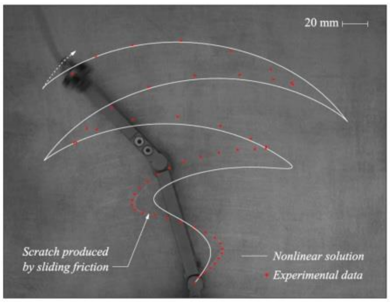

Fig. 2 A Ziegler pendulum traces the blowing-up motion of flutter induced by dry friction

Configurational forces occur in all situations where a solid body can change configuration through a release of energy. Examples of these forces are the Peach-Koehler interactions between dislocations or the forces acting on a phase boundary during phase transformations. Speaking of elastic rods, is it possible to find configurational forces acting on these? Theoretical and experimental proofs of the existence of these forces have been recently given [5], see Fig. 3.

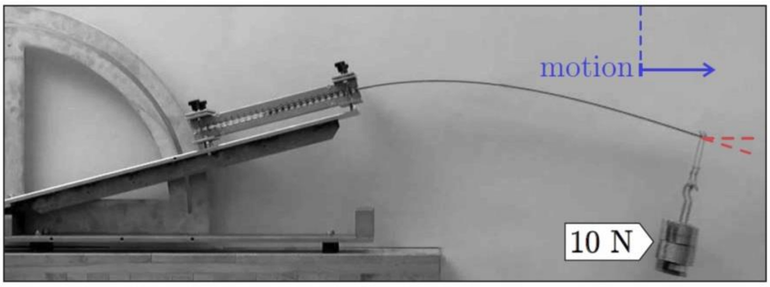

Fig. 3 An elastic rod subject to a dead weight (10 N), is ejected from a frictionless inclined constraint.
The experiment proves the existence of a configurational force.

Once discovered, we have been able to design systems exhibiting various configurational forces. These are shown to deeply influence stability [6] and have inspired us the design of a new type of scale, in which both equilibrium (as in a rigid arm balance) and deformation (as in a spring balance) determine the solution of a highly nonlinear system that can be calculated and realized to measure weight [7], see Fig. 4 (left).

Fig. 4 The elastica arm scale (left) and the torsional gun (right)

Configurational forces can also be induced by torsion, a concept which has inspired to us the idea of a new type of torsional actuator, nicknamed 'torsional gun' [8], Fig. 4 (right).

Finally, since the Euler's differential equation of the elastica governs an oscillating pendulum, a buckling rod, and a pendant drop, we pose the problem of the dripping of an elastic rod, namely: can an elastic rod subject to a transversal force self-encapsulate and take the shape of a drop? This problem can be analytically solved and experimentally attacked.

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