UNIVERSITY OF TRENTO - Italy
Department of Civil, Environmental and Mechanical Engineering

HOTBRICKS IAPP EU PROJECT

erc European Research Council

A boundary integral technique for the dynamics of a shear band

D. Giarola[1], D. Bigoni[1], D. Capuani[2]

[1]Department of Civil, Environmental & Mechanical Engineering, University of Trento
[2]Department of Architecture, University of Ferrara

Abstract

A shear band in an infinite, non-linear elastic and incompressible body, prestressed with an homogeneous initial deformation is considered. A boundary integral formulation (BEM-Boundary Element Method) has been developed to obtain the displacement field for the incremental problem of time-harmonic Green's functions.
Numerical simulations for a J2-deformation theory show for each state of prestress/anisotropy, a peak in the deformation of the shear band surface, that could represent a sort of resonance. In particular for different inclinations of the shear band with respect to the orthotropy axes, it is shown that if the inclination is equal to that of the real propagation of the band, the amount of the displacement in the middle of the shear band is maintained for very high frequencies.

Green's functions

Shear bands are localized deformations that are the preferential near-failure deformation mode of ductile materials. These emerge in a solid where a discontinuity in the velocity field appears. Following the model introduced by Bigoni e Dal Corso [1], a shear band can be represented with two surfaces fixed together with a set of zero thickness hinged quadrilaterals, so the material can only freely slide (Fig. 1).

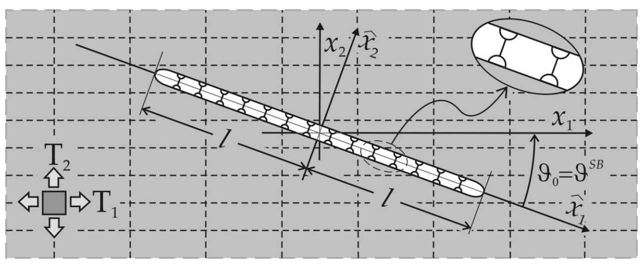

Figure 1: Schematization of the shear

Considering a perturbative approach is possible to represent the velocity field by using the time-harmonic Green's functions, found by Bigoni e Capuani [2], employed as a dynamic perturbation.

$$v^g(\mathbf{y}) = -\int_{\partial B} \left[[\![\dot{t}_{ij}]\!] n_i\, v_j^g(\mathbf{x},\mathbf{y}) - \dot{t}^g_{ij}(\mathbf{x},\mathbf{y}) n_i(\mathbf{x})\, [\![v_j]\!] \right] dl_x, \tag{1}$$

where v^g is the Green's displacement, v_j is the displacement of the shear band surface, $\dot{t}^g_{ij}$ is the Green's traction and n_i the normal versor on the shear band surface. The double brackets $[\![\cdot]\!]$ denote the jump of the relevant argument across the shear band. The boundary conditions on the sliding surfaces are:

- $\dot{t}_{21}=0$ null incremental nominal shearing tractions
- $[\![\dot{t}_{22}]\!]=0$ continuity of the incremental nominal normal traction
- $[\![v_2]\!]=0$ continuity of normal incremental displacement

then it's possible to explicit the equation of the displacement field and the constitutive equation:

$$v^g(\mathbf{y}) = \int_{\partial B} \dot{t}^g_{ij}(\mathbf{x},\mathbf{y}) n_i(\mathbf{x})\, [\![v_j]\!]\, dl_x, \tag{2}$$

$$\mathbf{n}\cdot\dot{\mathbf{t}}\,\mathbf{s} = n_l\, s_m\, \mathbb{K}_{lmkg} \int_{\partial B} \dot{t}^g_{ij,k}(\mathbf{x},\mathbf{y}) n_i(\mathbf{x})\, [\![v_j]\!]\, dl_x, \tag{3}$$

The Boundary Element Method

The BEM uses equation (3) to find the jump $[\![v_j]\!]$ applying unitary tractions on the shear band boundary. The discretizazion is performed on the shear band boundary and the displacement is represented with linear shape functions, then equation (3) becomes:

$$\mathbf{n}\cdot\dot{\mathbf{t}}\,\mathbf{s} = n_l\, s_m\, \mathbb{K}_{lmkg} \sum_{e=1}^{n} l_e \int_0^1 \dot{t}^g_{ij,k}(\mathbf{x},\mathbf{y}) n_i(\mathbf{x}) \left[[\![v_j^1]\!](1-\xi) + [\![v_j^2]\!]\xi \right] d\xi, \tag{5}$$

To validate the method, is possible to compare the numerical results with the analitical solution found by Bigoni & Dal Corso [1] for the quasi-static case.

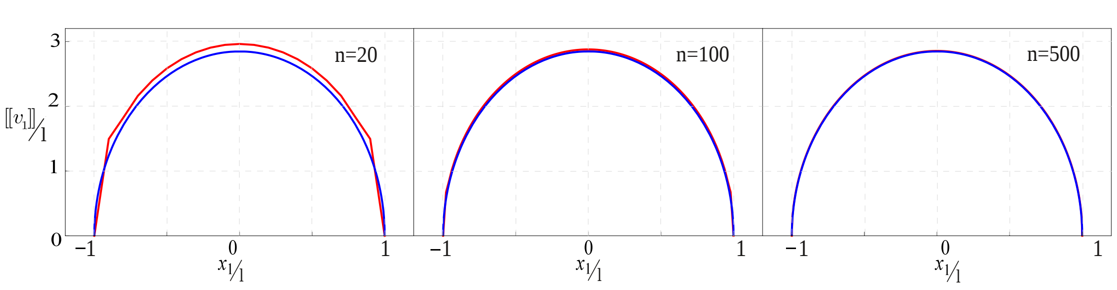

Figure 2: Displacements of the shear band surface for different discretization of the BEM. In red the numerical solution and in blue the analitical solution.

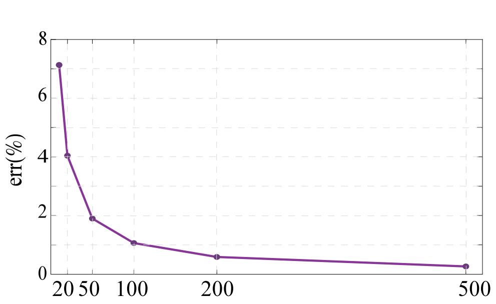

Figure 3: Percentual error of the displacement between the analitical and the numerical solution in the middle of the shear band. For a discretization with 500 elements, the error estimated is 0.26%.

Numerical results:

The frequency Ω depends on the lenght l of the shear band, an anisotropy parameter c and the wave lenght λ:

$$\frac{\Omega l}{c} = \frac{2\pi\, l}{\lambda} \tag{6}$$

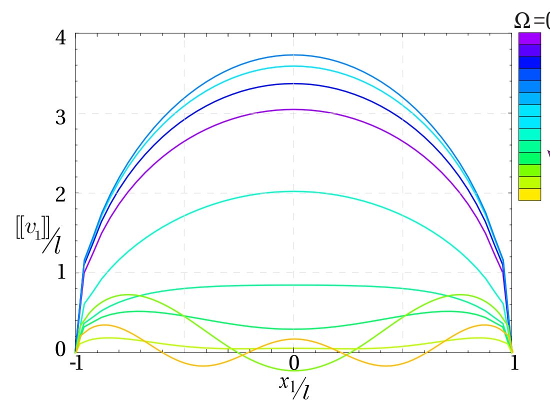

Figure 4: Jump of the displacement $[\![v_1]\!]$ along the shear band surface for increasing frequencies Ω. For low frequencies $[\![v_1]\!]$ has a simil-parabolic trend with maximum value in corrispondence of the middle of the shear band, but for higher frequencies it becomes a simil-sinusoidal trend.

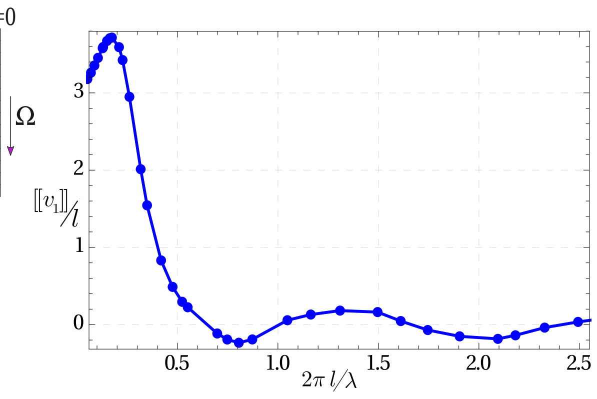

Figure 5: Jump $[\![v_1]\!]$ in the middle of the shear band (x_1=0), at the vary of the frequency. Starting from the quasi-static case (null frequency) the trend shows an increasing of the displacement up to a 23% where it reaches a peack that could represent a sort of resonance. After, it decades moving to the quasi-sinusoidal trend.

Below is considered a pre-existing shear band with different inclination with respect to the orthotropy axes.

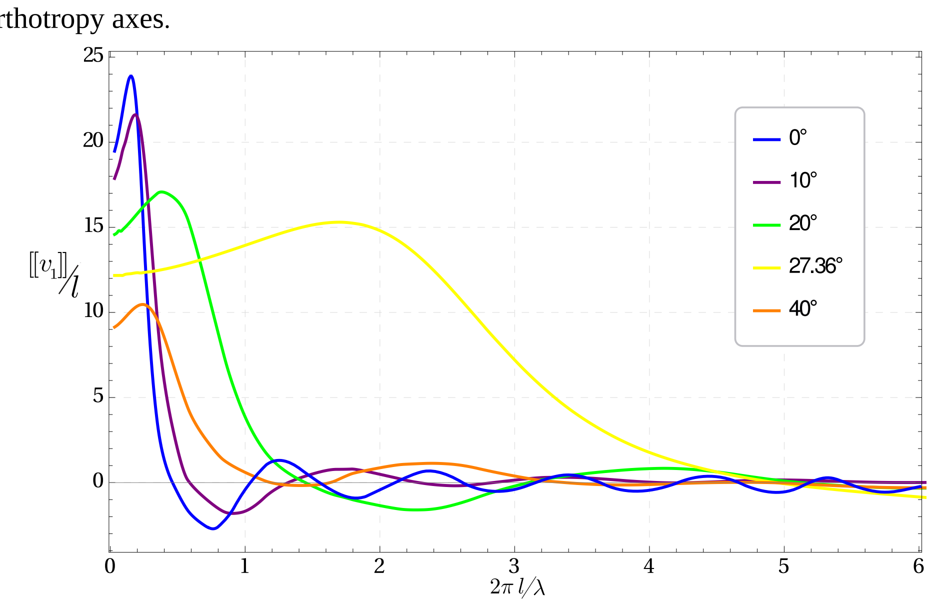

Figure 6: Displacement of the central element of the shear band surface at the vary of the frequency for different inclination of the shear band .

From Fig. 6 it can be noted that the maximum displacement is achieved for a null shear band inclination, corresponding to the condition when the band is orthogonal to the direction of wave propagation. Although the maximum displacement decreases at increasing value of the band inclination, a different trend appears for an angle of 27.36°, for which the amplitude of the displacement is mantained for very high frequencies with respect to the other cases. Such an inclination is known from quasistatic analysis to be the shear band propagation angle, depending on the anisotropy parameter through the equation:

$$\cot^2 \vartheta_0 = \frac{1 + 2\,\mathrm{sign}(k)\sqrt{\xi(1-\xi)}}{1-2\xi} \tag{7}$$

Once the jump in the displacement is known, the displacement field can be obtained through equation (2).

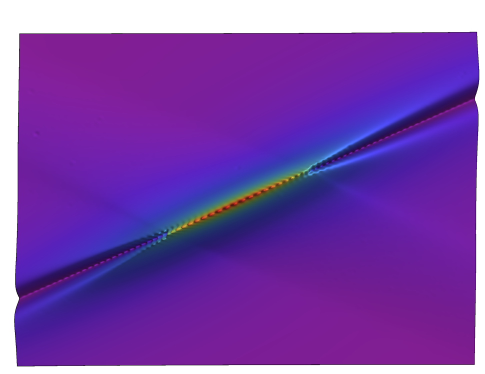

Figure 7: Displacement field of the shear band for a quasi-static case.

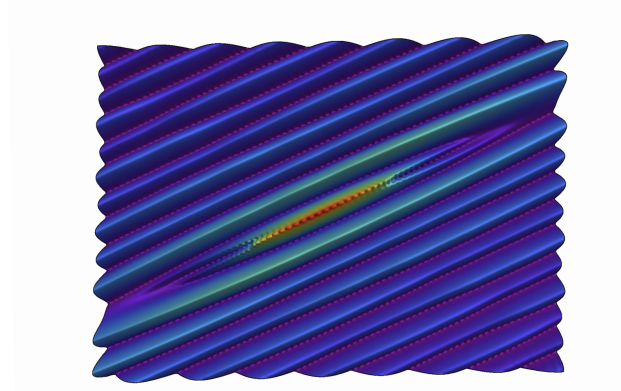

Figure 8: Displacement field of the shear band for a dynamic case with wave lenght of λ=2 l.

References

[1] D. Bigoni, F. Dal Corso (2008) The unrestrainable growth of a shear band in a prestressed material. Proceedings of the Royal Society A, 464, 2365-2390..

[2] D. Bigoni, D. Capuani (2005) Time-harmonic Green's function and boundary integral formulation for incremental nonlinear elasticity: dynamics of wave patterns and shear bands. Journal of the Mechanics and Physics of solids 53 (2005) 1163–1187.

Acknowledgements

Support from the ERC Advanced Grant 'Instabilities and nonlocal multiscale modelling of materials' FP7-PEOPLE-IDEAS-ERC-2013-AdG (2014-2019) is gratefully acknowledgements.