Folding of elastic solids & snaking, dripping, and fluttering of elastic rods

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How deformation patterns emerge during extreme loading of an elastic solid or an elastic structure? Buckling and instability has been advocated as a possible explanation of morphogenesis, describing the emergence of brain convolutions, of geological structures, and of the undulations of sea shells. Folding is a process in which bending localizes into sharp corners separated by almost undeformed elements. This process is rarely encountered in nature and is difficult to be described within the realm of the Cauchy theory of elasticity. On the other hand, it is shown that folding can be understood as a constitutive instability of a constrained-Cosserat elastic material occurring at the elliptic boundary [1, 2].

Using the elastica theory, configurational or 'Eshelby-like' forces are shown to arise in elastic structures when a change in configuration is possible, with a related release of energy. This concept has been developed in a series of recent works involving: a clamped elastic rod forced to slip inside a sliding sleeve [3], the 'dripping of an elastic rod' [4], the 'torsional locomotion' [5] and serpentine motion within a smooth channel [6].

The dynamics of an elastic rod in a cantilever configuration and subject to a tangential follower load of the 'Ziegler type' at its end (the 'Pfluger problem') is finally addressed. This structure is subject to a Hopf bifurcation, corresponding to the initiation of the 'flutter instability'. A new experimental set-up has been designed, produced and tested to realize the follower load. Experiments provide the evidence of flutter and divergence instability and provide the first proof that damping sources have a destabilizing effect on the system (the so-called 'Ziegler paradox').

From left to right: a snake in a rigid and frictionless channel, dripping and fluttering of an elastic rod

References

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