UNIVERSITY OF TRENTO - Italy
Department of Civil, Environmental and Mechanical Engineering

INSTABILITIES

BUCKLING OF THIN ELASTIC CYLINDERS

Gabriel Rossetto[1], Roberta Springhetti[1], Davide Bigoni[1]

[1]DICAM, University of Trento, Via Mesiano 77, 38123, Trento, Italy

Abstract

New results for buckling of thin elastic cylindrical shells are presented. In particular, a better insight into bifurcation of the primary equilibrium path for a cylinder under axial tension is offered. We derived the analytical condition of instability for a hollow cylindrical shell under axial tension according to a Lagrangian continuum framework, where the material has been assumed to be hyperelastic, incompressible and transversely isotropic about the axis of symmetry, with J2-deformation theory constitutive model.

Formulation of the problem

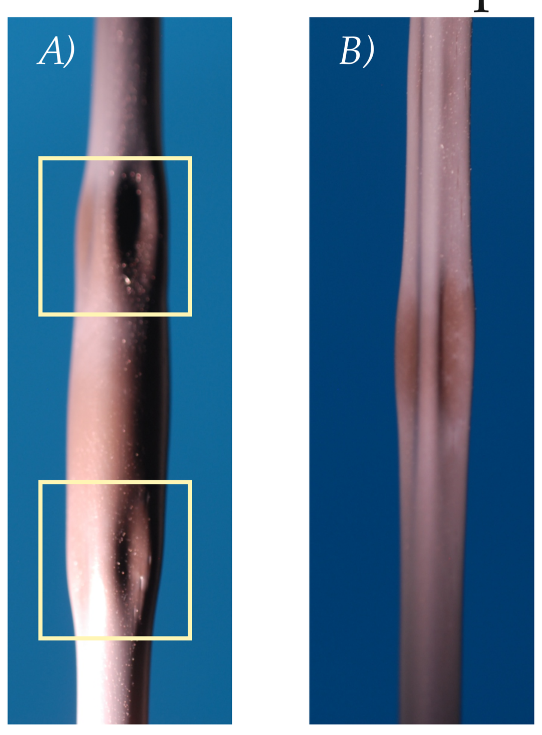

Fig. 1: Buckling under tension. A) Strain localisation and dimple initiation and B) dimple propagation. (Courtesy of the Laboratory for Physical Modelling of Structures and Photoelasticity, UniTn)

The experimental evidence clearly shows the existence of secondary equilibrium paths for a thin cylinder subjected to axial tension. The compressive hoop stresses induced by the reduction of the radius lead eventually to the formation of axisymmetric dimples (Fig. 1A). If the load is further increased, the dimples grow and merge together (Fig. 1B).
To analitically model the phenomenon, the hollow cylinder of Fig. 2 has been considered. The problem has been formulated within a Lagrangian continuum framework, where the incremental equilibrium condition reads (cp. [1,2]):

$$\mathrm{div}[\dot{\underline{\mathbf{S}}}] = \mathbf{0}, \tag{1}$$

and $\dot{\underline{\mathbf{S}}}$ is the increment of the first Piola-Kirchhoff stress tensor. Traction-free internal and external lateral surfaces have been considered as suitable boundary conditions, while perfect contact with rigid surfaces has been modeled for the upper and lower faces. Expressing the stress components as functions of the axial stretch λ_z and material constants (assumed the material to be hyperelastic and isotropic about the axis of symmetry), leads to the classical bifurcation condition:

$$\det(\underline{\mathrm{M}}) = 0, \tag{2}$$

where $\underline{\mathrm{M}}$ represents the matrix of the coefficients of the homogeneous system of equations governing the problem, which depends on material parameters as well as on the buckling parameters $\eta=k\pi/L$ (axial wave number, k=1,2,...) and n (circumferential wave number, n=0,1,2,...). The wave numbers η and n fully define the bifurcation mode.

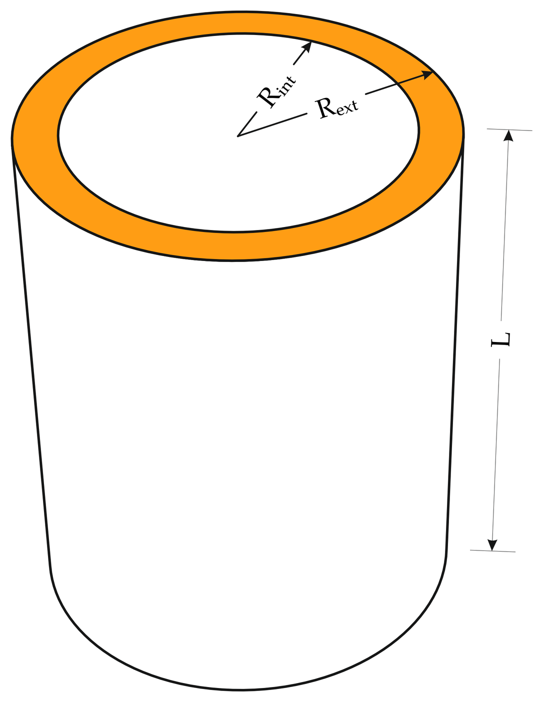

Fig. 2: Geometry of the system.

Results for J2-deformation theory material

For J2-deformation theory materials, the constitutive equation for uniaxial tension (or compression) reads:

$$\sigma = K\varepsilon_e^{N-1}\ln(\lambda_z), \quad \varepsilon_e = |\ln(\lambda_z)|, \tag{3}$$

where λ_z is the axial stretch, while the stiffness parameter K has been chosen equal to 0.1. Hence, the stability condition (2) depends only on λ_z and the wave numbers n and η.

Fig. 3 shows the critical stretch corresponding to instability for a particular value of the hardening exponent N (N=0.1) and for a specific ratio $\rho=R_{ext}/R_{int}$ (ρ=1.05, i.e. a very thin shell). The results are expressed in terms of the dimensionless quantity ηR_{int}: the smaller ηR_{int} the more slender the cylinder. Each curve was obtained for a different value of n (circumferential wave number). Finally, their lower envelope has been calculated, representing the possible combination of critical conditions for the instability of the shell. The curves are restricted to the interval $1.0<\lambda_z<2.234$: the lower bound reflects the fact that the cylinder is under tension, while the upper bound represents the value above which ellipticity is lost. Ellipticity loss corresponds to the development of shear bands, a phenomenon that we are not investigating.

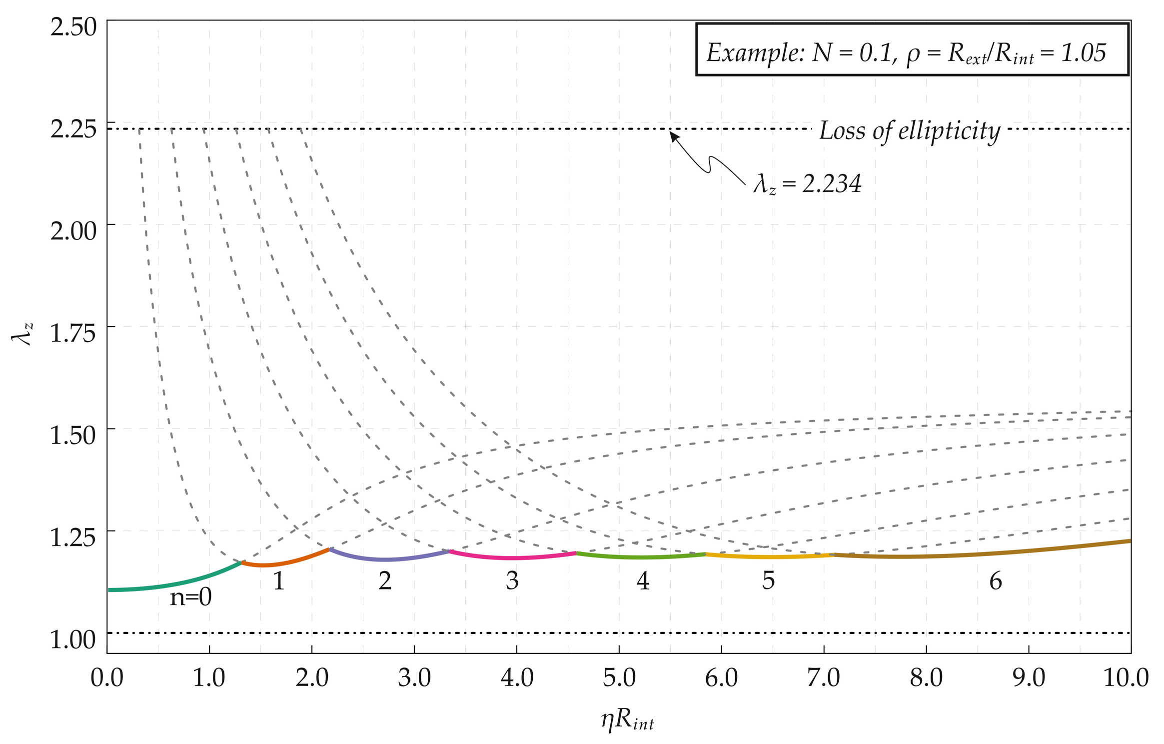

Fig. 3: Critical stretch for K=0.1, N=0.1 and ρ=1.05 for different values of circumferential wave numbers n as a function of ηR_{int} ($\eta=k\pi/L$, k=1,2,3,..) and lower envelope.

Fig. 4 extends the results to different values of hardening exponent N and ratio ρ. To simplify the interpretation of the diagram, curves sharing the same ratio ρ have been depicted with the same color in order to highlight the role of the hardening exponent N. For a fixed value of ρ, to a higher value of N corresponds a higher critical stretch. On the contrary, for a fixed value of N, high critical stretches are reached for very thick cylinders.

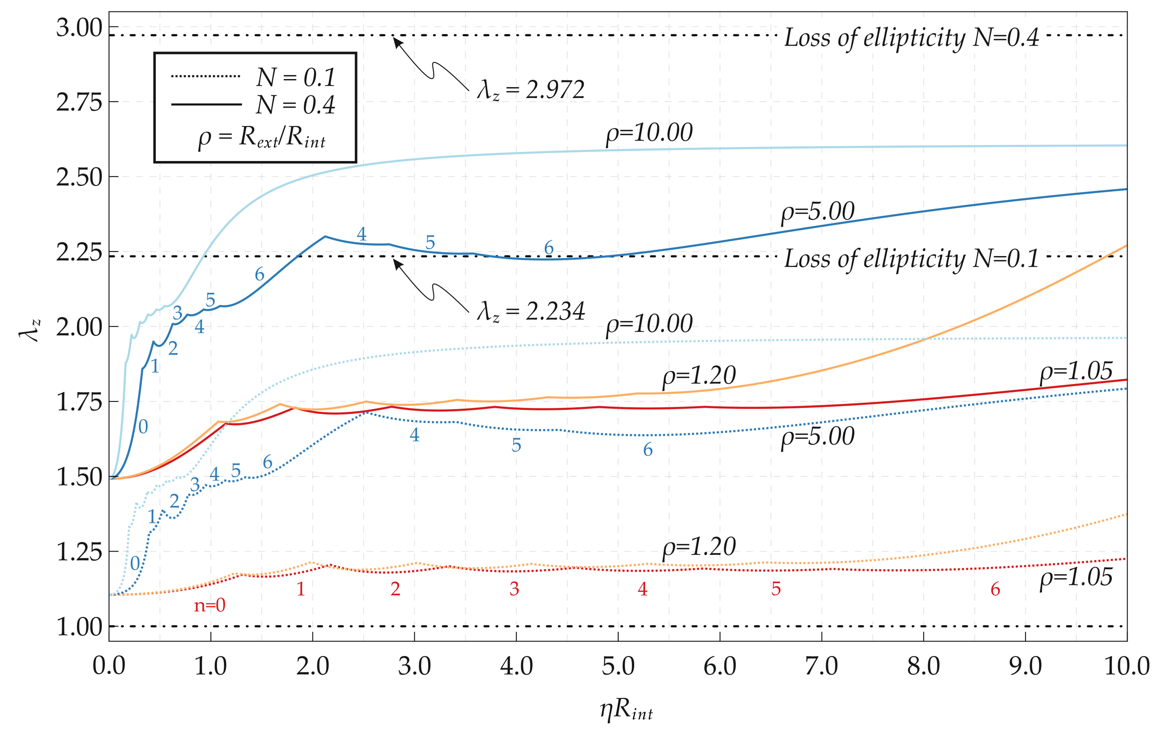

Fig. 4: Lower envelopes of the critical stretch for K=0.1, different values of the hardening exponent N and different ratios ρ as a function of ηR_{int} ($\eta=k\pi/L$, k=1,2,3,..).

References

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[2] Bigoni, D. (2012). *Nonlinear solid mechanics, bifurcation theory and material instability.* Cambridge, UK: Cambridge University Press.

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