Dispersion, localization and standing modes of in-plane Floquet-Bloch waves in Rayleigh beam lattices

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The aim of the present work is to investigate the dynamic response of a beam lattice for free and forced in-plane vibrations, with specific attention to dispersion degeneracies and localization patterns. A simple square grid-like lattice is considered to show that the vector problem of propagation of in-plane Floquet-Bloch waves involves more complex and unexpected behaviour than the scalar out-of-plane problem (analyzed in [1]).

By employing the exact time-harmonic solution on each beam, the analysis of free vibrations is addressed analytically, for both Euler-Bernoulli and Rayleigh beams. The computation of the dispersion relation clearly demonstrates the vector nature of the problem, as the structure of the dispersion surfaces results to be strongly dependent on the slenderness of the beams. The comparison between the dispersion surfaces for Euler-Bernoulli and Rayleigh beams shows that the effect of the rotational inertia is the overall lowering of the frequencies of propagation, with the exception of the pure axial waves that remain unaffected. For the Rayleigh beam lattice, we analyze the waveforms associated with the multiple Dirac cones connecting the two acoustic branches as well as identify peculiar standing modes characterized by a pure axial or pure flexural motion of the entire lattice.

The problem of forced vibrations is solved numerically (FEM) for different types of pulsating point loads, namely two orthogonal in-plane forces and an out-of-plane couple. The resulting lattice response is analyzed as follows: (i) the Fourier transform is used to represent the displacement field in the wave vector space, hence providing a clear description of which eigenmodes of the lattice are excited by the applied loads, (ii) the energy flow vector field is computed to evaluate the transmission properties of the lattice at different frequencies.

By means of this comprehensive analysis, we are able to identify the frequencies producing dynamic localizations. Specifically, the lattice exhibits unidirectionally localized modes or star-shaped wave patterns depending on the frequency and the type of applied load. A case of near-resonance vibrations is also identified at a frequency corresponding to a stationary point of the lowest dispersion surface where the presence of purely rotational standing waves prevails. Furthermore, to complete the comparison with the results obtained in [2] for the out-of-plane problem, we analyze a structured interface with enhanced rotational inertia that reveals a frequency-dependent behaviour exhibiting neutrality, pure reflection or wave trapping.

References

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