Mindlin second-gradient elastic properties from hexagonal lattice

Gianluca Rizzi, Daniele Veber, Francesco Dal Corso, Davide Bigoni
DICAM, University of Trento, Trento, Italy
E-mail: gianluca.rizzi@unitn.it, daniele.veber@unitn.it,
francesco.dalcorso@unitn.it, davide.bigoni@unitn.it

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Micromechanical effects induced by the presence of heterogeneities and microstructures within solids can be described through nonlocal and high-order models.

With reference to an hexagonal Representative Volume Element (RVE), a refined description for the effective behaviour of an infinite planar and periodic lattice is investigated, Fig. 1. The bars composing the RVE are considered connected to each other through relative hinges and characterized by three different values of stiffness.

Closed form expressions for the properties of the homogeneous Mindlin second-gradient elastic material [2] equivalent to the lattice are obtained through energy comparison under the boundary condition of generic (but self-equilibrated) quadratic displacement. The effective response of isotropic Cauchy material is retrieved [1] when the boundary condition is restricted to the linear case. The effective parameters for the Mindlin second-gradient elastic solid are obtained by imposing the energy matching between the lattice and the equivalent solid under the generic quadratic boundary condition. Such a nonlocal behaviour is found to be non-isotropic and in particular corresponding to a special class of anisotropy. It follows that in general a microstructure leading to isotropic local behaviour may display anisotropy in the nonlocal effects.

The reliability of proposed model is assessed through comparison in the mechanical response between the lattice and the equivalent higher-order solid, predicted with a developed Finite Element Code, under various boundary value problems.

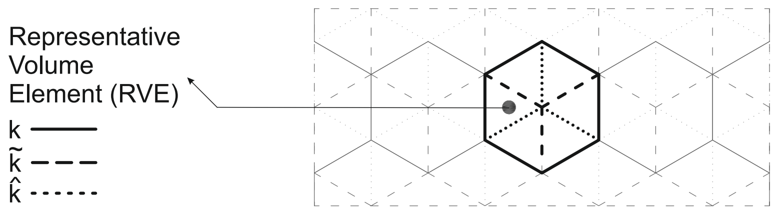

Figure 1: Infinite lattice realized as the periodic repetition of an Hexagonal Representative Volume Element (RVE), composed by bars with different values of stiffness, k, $\widetilde{\text{k}}$, $\widehat{\text{k}}$.

References

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[2] *Mindlin, R.D., Micro-structure in linear elasticity, Archs ration. Mech. Analysis,* **16***, page 15-78 (1964).*

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