Wave Scattering by Arrays of Shear Bands

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Abstract. A plane-strain model of multiple shear bands, arranged in different configurations, is presented in order to investigate the effects of their dynamic interaction. Reference is made to a material stressed to the verge of instability and subject to incoming harmonic waves of small amplitude. It is shown that shear band arrays may be subject to resonance and corresponding shear band growth or, conversely, to shear band annihilation. At the same time, multiple scattering may bring about focusing or, conversely, shielding from waves.

Keywords: Shear band · Wave propagation · Pre-stress

1 Introduction

Interaction of shear bands has been documented so far for quasi-static deformation processes [1], where it has been shown that different shear band patterns emerge as related to load conditions and material properties of the samples, and where parallel, aligned, and converging shear bands [2] are frequently observed. In dynamics, results are restricted to high strain-rate loading, where numerical simulations [3] have been presented. In this context, experiments on metallic glass [4] show the development of a complex texture of multiple shear bands, with complex interactions.

Direct experimental investigation on the fine development of shear bands in a material and their effect on the stress field during time-harmonic vibrations remains difficult to be carried out, so that mechanical modelling represents the worthwhile way to shed light on a complex phenomenon, whose comprehension is a key point for engineering materials with enhanced mechanical properties.

In this paper, shear bands of finite length are idealized as discontinuity surfaces, formed inside the infinite medium at a certain stage of a continuous deformation. Each shear band is seen as a weak surface whose faces can freely slide, but are constrained to remain in contact.

2 Constitutive Equations

The incremental behaviour of an infinite, incompressible, nonlinear elastic material, homogeneously deformed under plane strain condition, is considered. According to Biot [5], the constitutive relations between the nominal stress increment $\dot{t}_{ij}$ and the

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gradient of incremental displacement $v_{i,j}$ (a comma denotes partial differentiation) can be expressed in the principal reference system of Cauchy stress (here denoted by axes x_1 and x_2) as follows

$$\dot{t}_{ij} = \mathrm{K}_{ijkl} v_{l,k} + \dot{p}\delta_{ij} \tag{1}$$

where repeated indices are summed and range between 1 and 2, δ_{ij} is the Kronecker delta, $\dot{p}$ is the incremental hydrostatic stress and K_{ijkl} are the instantaneous moduli. These moduli possess the major symmetry $\mathrm{K}_{ijkl} = \mathrm{K}_{klij}$ and are functions of principal components of Cauchy stress, σ_1 and σ_2, describing the pre-stress, and of two incremental moduli μ and μ_* (which can depend arbitrarily on the current stress and strain) corresponding to shearing parallel to, and at 45° to, the principal stress axes. The non-null components are:

$$\mathrm{K}_{1111} = \mu_* - \frac{\sigma}{2} - p, \quad \mathrm{K}_{1122} = \mathrm{K}_{2211} - \mu_*, \quad \mathrm{K}_{2222} = \mu_* + \frac{\sigma}{2} - p \tag{2}$$

$$\mathrm{K}_{1212} = \mu + \frac{\sigma}{2}, \quad \mathrm{K}_{1221} = \mathrm{K}_{2112} = \mu - p, \quad \mathrm{K}_{2121} = \mu - \frac{\sigma}{2}$$

with

$$\sigma = \sigma_1 - \sigma_2, \quad p = (\sigma_1 + \sigma_2)/2. \tag{3}$$

Equation (1) is complemented by the incompressibility constraint for incremental displacement v_i

$$v_{i,i} = 0. \tag{4}$$

Constitutive Eqs. (1)–(4) describe a broad class of material behaviors, including all possible elastic incompressible materials which are isotropic in an initial state, but also materials which are orthotropic with respect to the principal stress directions.

3 The Boundary Value Problem

In Fig. 1 possible different arrays of shear bands, each one of total length $2l$, are represented together with local reference systems $(\hat{x}_1, \hat{x}_2)$ centered on each shear band, with $\hat{x}_1$-axis aligned parallel to the shear band, and rotated at an angle θ with respect to the principal reference system (x_1, x_2) introduced for constitutive Eq. (1).

According to the model described in [6], by introducing the jump operator for a generic function f, smooth on two regions labeled "+" and "−", and discontinuous across the surface S_n of the nth shear band, as

$$[\![f]\!] = f^+ - f^- \tag{5}$$

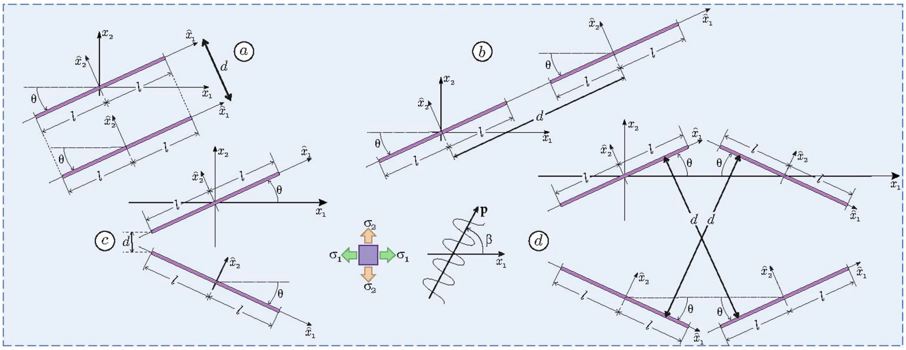

Fig. 1. Arrays of shear bands: **a** parallel; **b** aligned; **c** converging; **d** with 4 shear bands.

where $f^{\pm}$ denote the limits approached by function f at the faces of the discontinuity surface, the boundary conditions at shear band surface S_n can be written as

$$[\![\hat{v}_2]\!] = 0, \quad [\![\hat{t}_{22}]\!] = 0, \quad \hat{t}_{21} = 0 \tag{6}$$

with $\hat{v}_i$, $\hat{t}_{ij}$ being incremental displacement and incremental stress components in the local reference system.

Time-harmonic incident shear waves of circular frequency Ω characterized by incremental displacement field $\mathbf{v}^{inc}(\mathbf{x})$, with amplitude A and phase velocity c, propagation direction $\mathbf{p}$ and direction of motion $\mathbf{d}$, are considered

$$\mathbf{v}^{inc} = A\mathbf{d}e^{i\frac{\Omega}{c}(\mathbf{x}\cdot\mathbf{p}-ct)} \tag{7}$$

so that the total incremental displacement field $\mathbf{v}(\mathbf{x})$ is given by the sum of the incident and of the scattered field $\mathbf{v}^{sc}(\mathbf{x})$.

The dynamic response of the medium in terms of total incremental displacement field can be found by adopting integral representations for the wave-fields as is shown in [7], using the infinite body Green function [8]. The system of boundary integral equations in the unknown scalar functions $[\![\hat{v}_1]\!]$ at each S_n, i.e. the jumps of tangential incremental displacement across the faces of each shear band, has been given in [7].

4 Numerical Examples

Using a collocation method, the boundary integral equation system in [7] can be transformed into a linear algebraic system where the unknown nodal values of displacement jumps across shear band faces can be determined in terms of known nodal values of incident tangential tractions on shear bands. To this purpose, each shear band is subdivided into Q line elements (Q = 100), and a quadratic variation of the incremental displacement jump is assumed within each line element, with the exception of two line elements situated at the shear band tips, where a square root variation is adopted to take into account the singularity at the shear band tip.

A ductile low-hardening metal, modelled through the J_2-deformation theory of plasticity [9–11], with the hardening exponent N = 0.4 (representative of a medium carbon steel) is considered. A level of prestress close to the elliptic boundary, with $k = \sigma/2\mu = 0.87$ and $\xi = \mu/\mu_* = 0.26$, corresponding to shear band inclination $\theta \cong \pm 26°$, is assumed so that some shear bands are expected to be already formed.

The material response to shear waves with angle of incidence β (see Fig. 1) and wavenumber $\Omega l/c_1 = 1$ (c_1 is the propagation velocity in the direction of x_1-axis), is shown in terms of modulus of the incremental deviatoric strain field for arrays of shear bands which are parallel (Fig. 2), aligned (Fig. 3), converging (Fig. 4) or formed by

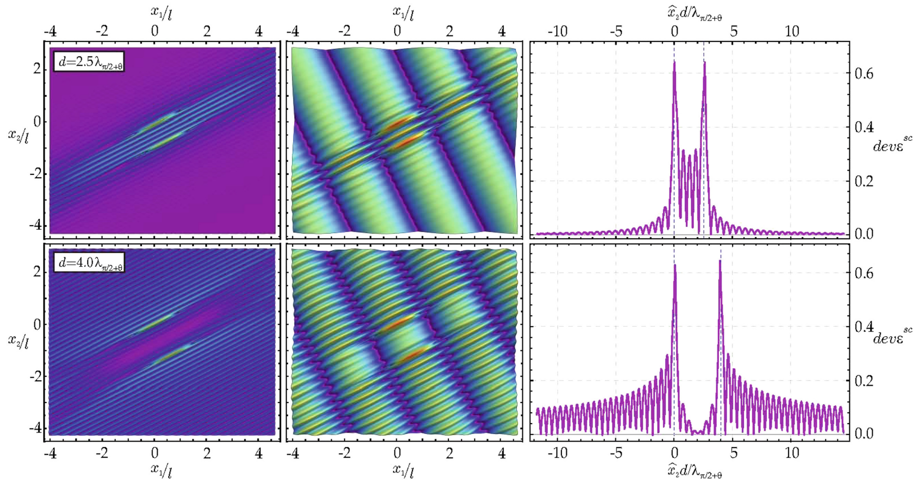

Fig. 2. Parallel shear bands with parallel incident wave (β = θ).

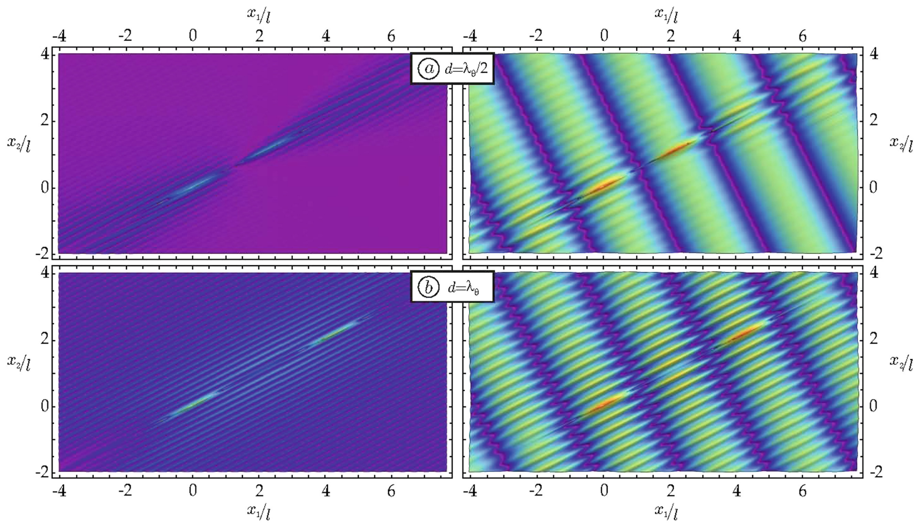

Fig. 3. Aligned shear bands with parallel incident wave (β = θ).

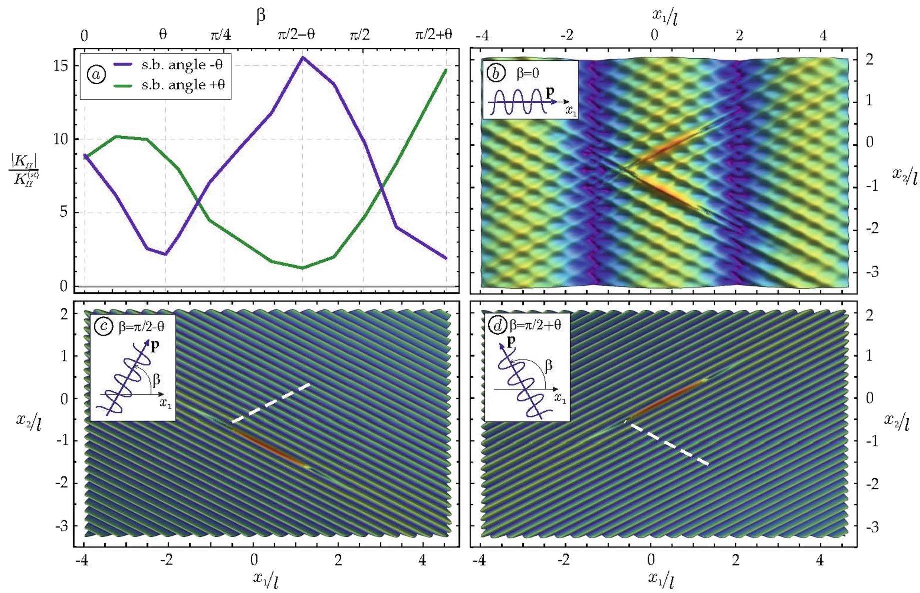

Fig. 4. Converging shear bands with different angles of wave incidence (distance $d = l/10$).

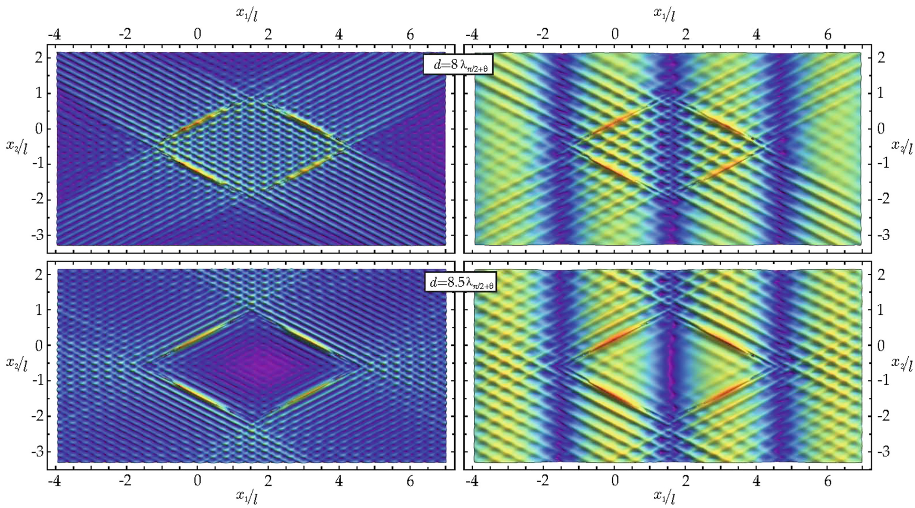

Fig. 5. Four shear bands with horizontal incident wave (β = 0).

four shear bands (Fig. 5). In all figures, maps show the scattered wave-field on the left hand side and the total wave-field on the right hand side, with the exception of Fig. 4 where only the total wave-field is reported.

In Fig. 2, graphs on the right side are cross-sections of the scattered deviatoric strain along $\hat{x}_2$-axis, at the shear band centre. The two cases differ only in the distance d between the shear bands, with $d = 2.5\lambda_{\pi/2+\theta}$ or $d = 4\lambda_{\pi/2+\theta}$, where λ_α is the wavelength in the propagation direction singled out by angle α. Analogously, in Fig. 3 where $\alpha = \theta$.

In Fig. 4a, the dimensionless stress intensity factor is reported as a function of the angle β of wave incidence. It can be seen that when the wave travels orthogonal to one shear band, the relevant shear band tip is unloaded. This effect, which corresponds to annihilation of a shear band, is visible in parts (c) and (d) of the same figure, where one shear band (dashed white line) "disappears", while the other one is "highlighted".

In Fig. 5, focusing of signal is noted in the area circumscribed by shear bands at a distance $d = 8\lambda_{\pi/2+\theta}$, whereas shielding is evidenced at a distance $d = 8.5\lambda_{\pi/2+\theta}$ producing an "island of stress relief".

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