Strain-gradient effects in the discrete/continuum transition via homogenization

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Abstract

Many materials existing in nature and man-made are characterized by a microstructure that can be represented through a, more or less, periodic spatially repetition of a unit-cell (RVE) (Fig.1). In order to model a heterogeneous material through a simple constitutive relation to be exploited in mechanical, thermal and electromagnetic problems, it is possible to introduce the concept of homogeneous continuous solid energetically equivalent to the heterogeneous solid. Imposing the equivalence in terms of the energy stored within the two solids when subjected to the same boundary conditions leads to the definition of the constituent's properties of the equivalent homogeneous material.

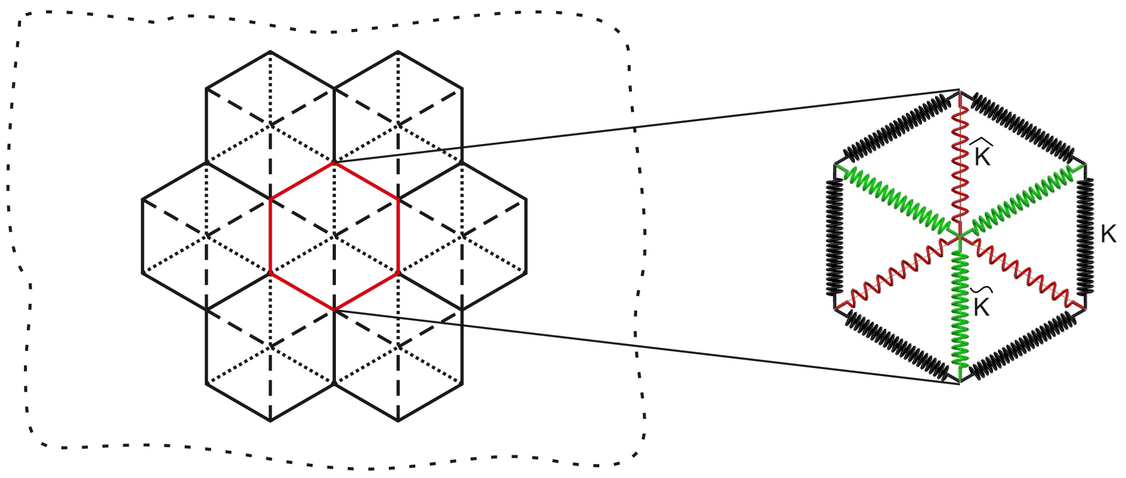

Figure 1: Periodic Lattice

However, an improved description for the equivalent homogeneous material can be achieved when higher-order constitutive models are considered, due to the additional kinematical degrees taken into account, as in the case of Mindlin material [1]. The study of a periodic lattice, flat, infinitely extended and composed of hexagonal cells formed by three orders of bars having different stiffness and linked to each other by relative hinges (Fig.1) is here presented. Results from the imposition of linear displacement boundary conditions confirms the local equivalent properties obtained by [2] with reference to a periodic pattern similar to that considered in a discretization problem. Our aim is to achieve the higher-constitutive parameters for the equivalent higher-order homogenous solid extending the boundary conditions to a quadratic displacement field, similarly to [3].

Introduction

An infinite periodic microstructured material is considered as the repetition within a plane of an hexagonal representative cell, formed by three orders of bars with different stiffness and which are linked to each other by relative hinges (Fig.2).

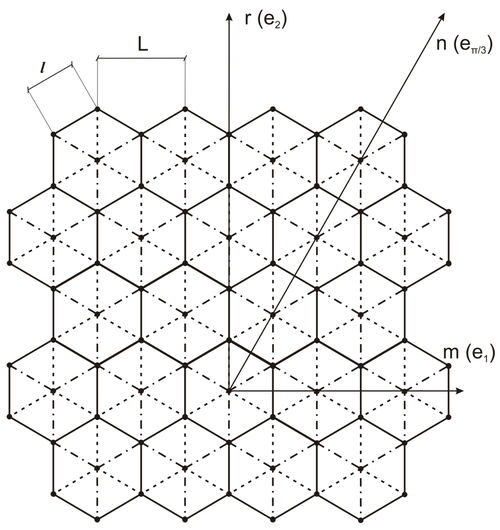

Figure 2: Lattice with reference system

The orthogonal axes m and r are introduced together with their respective unit vectors e_1, e_2. It is also instrumental for the computations to introduce the axis n, inclined of an angle $\pi/3$ with respect to the m axis. L ad l are respectively the distance between two contiguous center of RVE and the length of its side. Imposing on the hexagon hinges the boundary conditions expressed by the following generic quadratic displacement function

$$u_i = \alpha_{ij} \cdot x_j + \beta_{ijk} \cdot x_j \cdot x_k \tag{1}$$

(where u is the displacement vector that is applied to the RVE nodes, x is the position vector of the nodes, α_{ij} is a tensor of the second order is representative of the deformations and β_{ijk} is a tensor of the third order is representative of the curvatures) and comparing the stored energy within the lattice cell with that stored within an equivalent continuous material leads to the definition of the unknown constitutive equivalent parameters.

First order homogenization: linear displacement boundary condition

The energetic comparison provides, when linear boundary condition are applied ($\beta_{ijk} = 0$ in the Eq.(1)), the same results obtained in [2] and defining a first order effective isotropic material. In particular, the effective bulk and shear modulus (κ^{eff} and μ^{eff}) are expressed by:

$$\kappa^{eff} = \frac{K_1}{2\sqrt{3}l} \tag{2}$$

$$\mu^{eff} = \frac{3\sqrt{3}K_3}{4lK_2} \tag{3}$$

where

- $K_1 = k + \widetilde{k} + \widehat{k}$
- $K_2 = k \cdot \widetilde{k} + k \cdot \widehat{k} + \widetilde{k} \cdot \widehat{k}$
- $K_3 = k \cdot \widetilde{k} \cdot \widehat{k}$

The energetic comparison has shown the need of introducing an additional periodic displacement field in the boundary conditions in order to ensure the equilibrium of the lattice for any linear boundary condition.

Second order homogenization: quadratic displacement boundary condition

As a second step in the energetic comparison the boundary condition is extended to be the purely quadratic boundary condition ($\alpha_{ij} = 0$ in the (Eq.1)).
The additional energy provided by the high-order constants has to be compared with the Residual Energy (W_{res}), given by subtracting W_β to the energy of the lattice the energy

$$W_\varepsilon^\beta = \frac{1}{2}\varepsilon_{ij}^\beta E_{ijkl}^I \varepsilon_{kl}^\beta \tag{4}$$

in which the ε_{ij}^β are the strain generated by the quadratic displacement boundary condition and E_{ijkl}^I is the fourth order tensor obtained from the first order homogenization.
The expression of the W_{res} shows that the lattice behaves anisotropically at higher-order, and more specifically that a symmetry is present for multiple rotations of sixty degrees (Symmetry class Z6). It can be therefore concluded that higher-order materials with different classes of symmetry exists, namely, isotropic as local behaviour and anisotropic as non-local behaviour as effective response of the considered periodic lattice.

References

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