Snap-through instability of a soft robotic arm

Costanza Armanini[1], Francesco Dal Corso[1], Diego Misseroni[1], Davide Bigoni[1]

[1]Department of Civil, Environmental & Mechanical Engineering, University of Trento, Italy

Abstract

An inextensible cantilever beam undergoing large deflection has the free end loaded by a concentrated force and the clamped end continuously rotating. The modelling of this system through the Elastica allows for the analytical solution for the deflection of the beam at varying the clamp angle. To experimentally investigate the critical and post critical behaviour of the system, a new device, the Elastic Catapult, has been designed and realized. Tests performed at different loads show that the system suffers a snap-through instability when the applied load is greater than the Euler buckling load. The observed behaviour is theoretically predicted by the proposed analytical model, providing the critical angles for which the snap-through phenomenon occurs.

The elastica equation

We consider an elastic cantilever beam of total length l and stiffness B, subject to a concentrated load applied to its free end with an inclination angle α. The beam is assumed as inextensible and shear deformations are not considered.

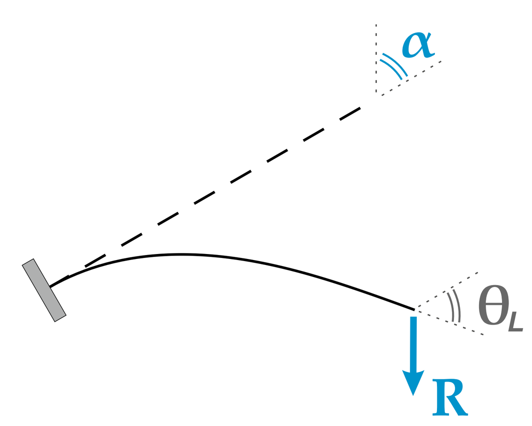


Figure 1: A sketch of the considered elastic structural system

The behaviour of the rod is described by a non linear, II order differential equation which is provided by the analysis of the kinematics of the beam and the standard linear constitutive law:

$$\begin{cases} B\theta''(s) + P_1 \cos\theta(s) + P_2 \sin\theta(s) = 0 \\ \theta'(l) = 0 \\ \theta(0) = 0 \end{cases} \qquad (1)$$

where P_1 and P_2 represents the vertical and the normal components of the applied load P, respectively.

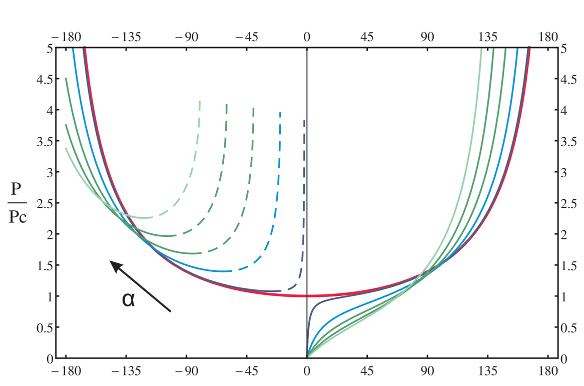
The equilibrium paths of the system

Through many mathematical steps, we find the closed form solution of equation (1), which provides the relationship between the applied load and the deflection of the beam, represented by θ_L, the rotation angle of the free end:

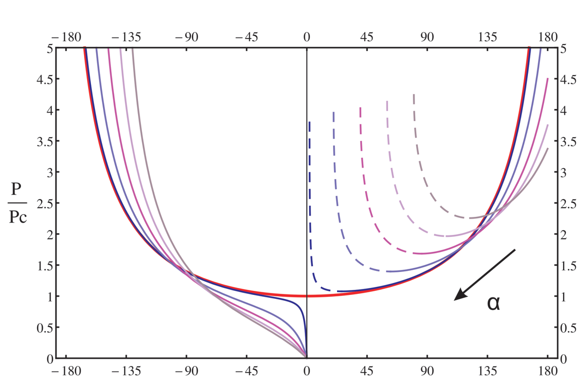
$$P = \frac{B}{l^2}[K(k) - K(\phi_0, k)]^2 \qquad (2)$$

where $\phi_0 = \arcsin\left[\frac{\sin(\frac{\alpha}{2})}{k}\right]$

and $K(k)$ and $K(\phi_0, k)$ are two elliptic integrals of the first kind, complete and incomplete respectively.
Equation (2) provides the equilibrium paths of the system:



(a) *Equilibrium paths for* $\alpha > 0$



(b) *Equilibrium paths for* $\alpha < 0$

Displacements and deformations

The analysis gets along with the displacements field, that leads to define the deformed configurations of the beam. Figure (2) shows some examples of the configurations obtained for different values of the inclination angle of the load, α:

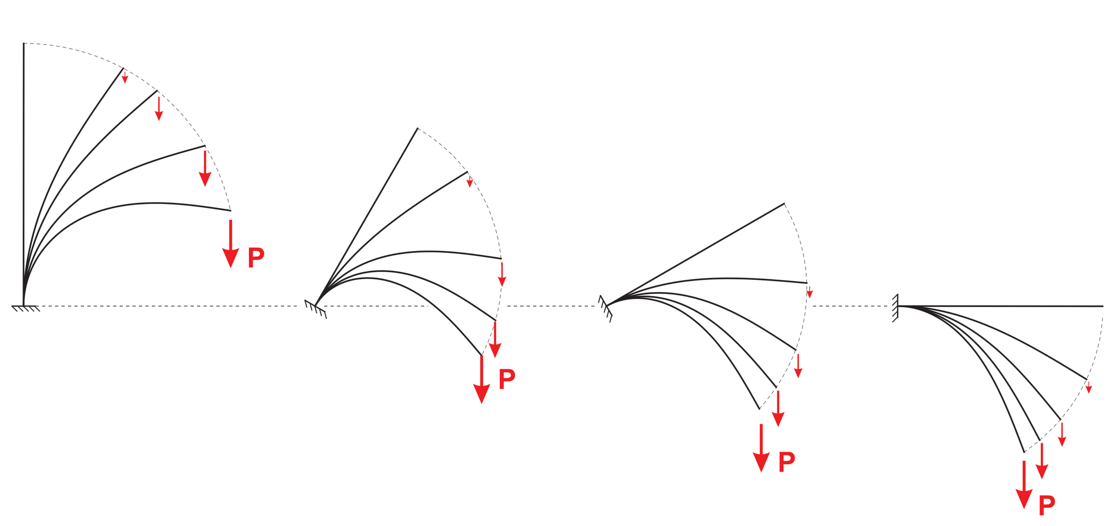


Figure 2: Deformed shapes

The elastic catapult

In order to study the critical and post-critical behaviour of the system, we designed a novel device, called the elastic catapult. The set up consists in a polycarbonate beam, constrained to a rotating fixed joint.

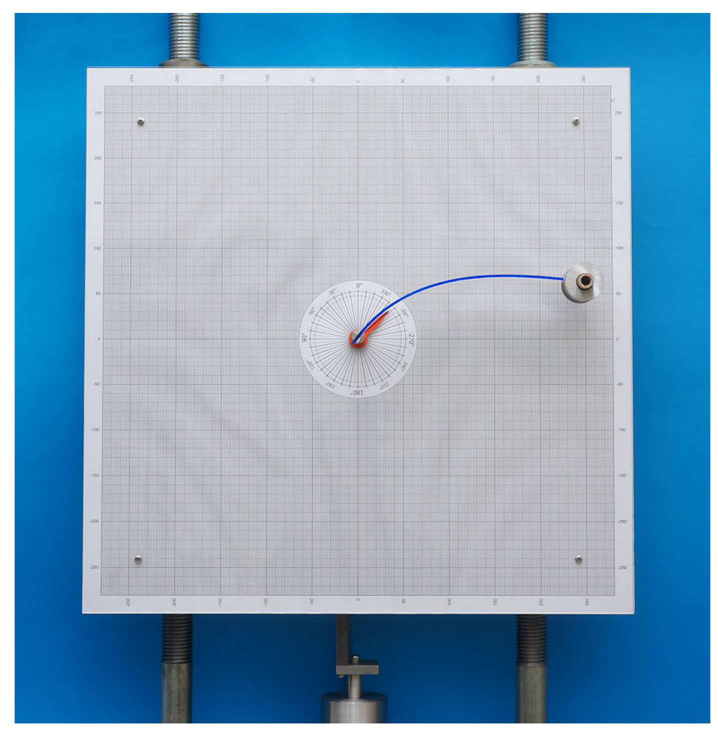
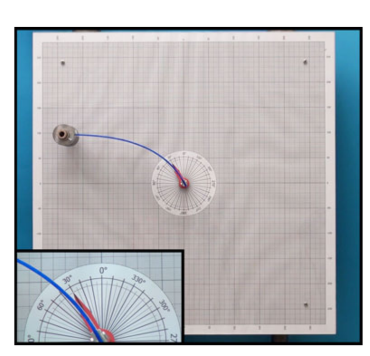


Figure 3: The Elastic Catapult as the experimental realization of the considered structural system

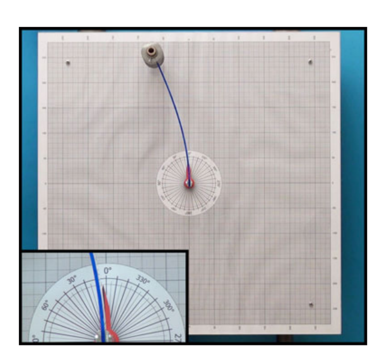
From an analytical point of view, the device simulates the behaviour of a cantilever beam subjected to a concentrated force, which continuously changes its application angle α.

Experimental tests for $P < P_{cr}$

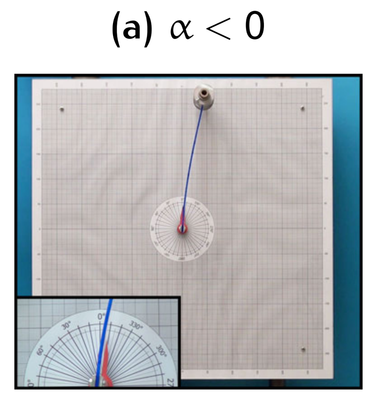
At first, some tests were run with an applied load which was lower than the Euler critical load of the beam. In this case, the system rotates without suffering instabilities and it gradually reverse its bending.



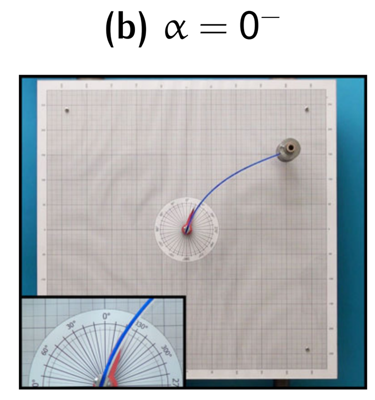
(a) $\alpha < 0$



(b) $\alpha = 0^-$



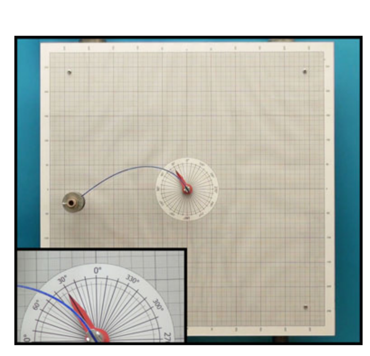
(c) $\alpha = 0^+$



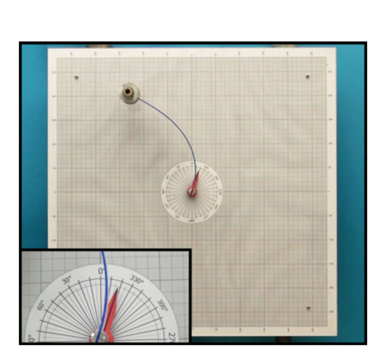
(d) $\alpha > 0$

Experimental tests for $P > P_{cr}$

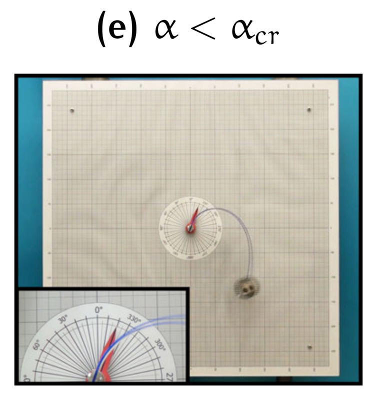
When the applied load is greater than the Euler critical load, the system display a snap-through phenomenon: at a specific "'critical"' value of the inclination angle of the beam, instability occurs and the system suddenly revers its bending.



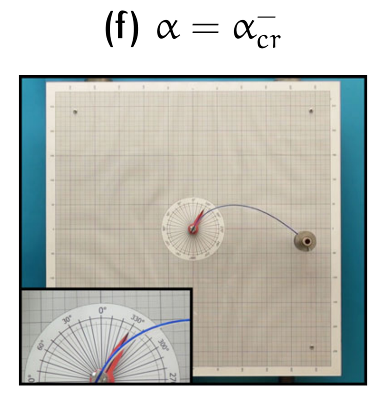
(e) $\alpha < \alpha_{cr}$



(f) $\alpha = \alpha_{cr}^-$



(g) $\alpha = \alpha_{cr}^+$



(h) $\alpha > \alpha_{cr}$

Theoretical explanation

The behaviour experimentally observed can be explained considering the equilibrium path provided by equation (2). Once the applied load to the free end of the beam is fixed, the path followed by the system for increasing values of α sees three different situations, as shown in figure (4)

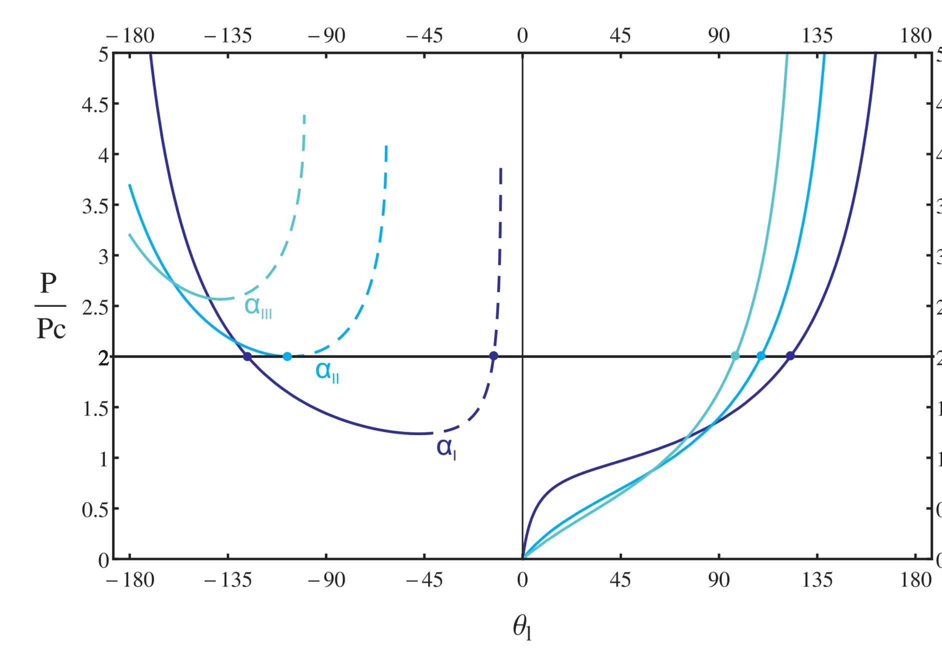


Figure 4: The path followed by the system

- when α is lower than α_{cr}, there are three different solutions to equation (2) and just as many intersections between the equilibrium curve of the system and the line that indicates the value of the applied load (see α_I);
- when α is equal to α_{cr}, there are only 2 solutions to equation (2), since the equilibrium path is tangent to the line of the applied load. These solutions are equal in value but with different sign (see α_{II});
- when α is higher than α_{cr}, there is only one solution to equation (2) and it corresponds to a positive value of the bending θ_L (see α_{III}).

Figure (5) shows a comparison between the theoretical analysis and the experimental test, which confirms the reliability of the model.

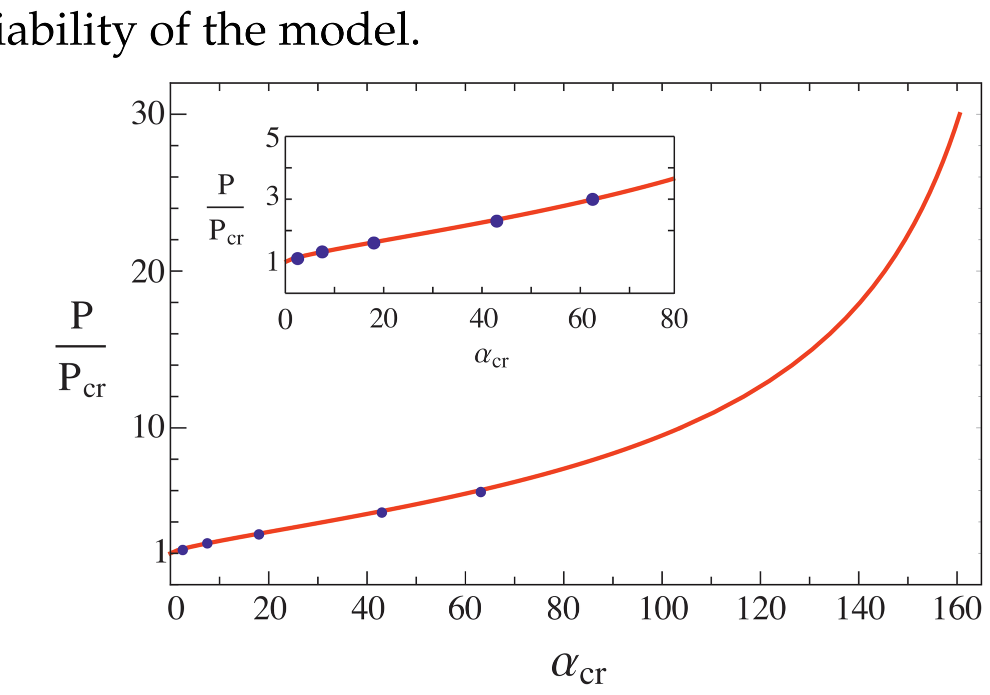


Figure 5: Comparison between theoretical and experimental results

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