WORKSHOP
DYNAMIC PHENOMENA IN MEDIA WITH MICROSTRUCTURE[1]

Book of abstracts

School of Mechanical Engineering
The Iby and Aladar Fleischman Faculty of Engineering
Tel-Aviv University

October 7–12, 2018
Tel Aviv, Israel

[1]Funded by ISF – Israel Science Foundation

Contents

About the workshop

One of the emerging fields in solid mechanics nowadays concerns a wide range of objects which their quasi–static and dynamic properties are based on discrete 2D and 3D lattice structures. Discreteness and periodicity in particular result in phenomena that cannot be adequately be described by continuum mechanics models. These topics became notably important in view of the progress in various areas of science: physics, chemistry, biology, acoustics, material sciences, and mechanical engineering.

In recent years, novel developments have been made in the area of functional equations, linked to lattice models in the context of high–frequency homogenization, fracture, scattering, and modelling of nano and metamaterials.

The methods in the dynamic phenomena in media with microstructure developed in certain areas can be effective in other areas, from point of view of the problems and the mathematical formulations. With this in mind, the main goal of the workshop is to bring together scientists from different areas. In this way, the achievements, current and prospective projects with the used approaches will be discussed.

The following topics are suggested (and not limited) to discuss:

- General dynamic effects in lattices
- Pattern dynamic formation
- Micro and nano aspects of lattice phenomena
- Two and three-dimensional lattices with defects
- Wave transmission and localization
- Wave guiding and cloaking
- Dynamic phase transition
- Ordered regimes of lattice dynamic transformation
- Dynamic splitting
- Lattice fracture
- Atomistic computer simulations
- Beam-made lattices
- Structure optimization

Committees

Scientific Committee

- Prof. Leonid Slepyan (Tel–Aviv University, Israel) – Co-chairman
- Prof. Dov Sherman (Tel–Aviv University, Israel) – Co-chairman
- Dr. Igor Berinskii – Scientific secretary
- Prof. Davide Bigoni, (University of Trento, Italy)
- Prof. Gennady Mishuris (Aberystwyth University, UK)
- Prof. Alexander Movchan (University of Liverpool, UK)
- Prof. Andrew Norris (Rutgers University, USA)
- Prof. Lev Truskinovsky (Ecole Polytechnique, France)
- Prof. Anna Vainchtein, (University of Pittsburgh, USA)

Organizing Committee

- Prof. Dov Sherman (Tel–Aviv University, Israel)
- Prof. Leonid Slepyan (Tel–Aviv University, Israel)
- Prof. Gennady Mishuris (Aberystwyth University, UK)
- Prof. Alexander Movchan (University of Liverpool, UK)
- Dr. Igor Berinskii (Tel–Aviv University, Israel)
- Dr. Liron Ben-Bashat Bergman (Tel–Aviv University, Israel)

Abstracts

Resonant waves and localization phenomena in lattices

Sagdulla Abdukadirov[1], Mark Ayzenberg[2]
[1] Tashkent Architectural Building Institute, Uzbekistan; [2]Ben Gurion University, Israel

Non-steady-state wave processes are numerically simulated in mass-spring lattices. Uniform 2D lattices of square, rectangular, triangular, hexagonal cells and 3D square-cell lattices are considered. Resonance regimes are explored for lattices excited by oscillating point force and the formation is revealed of star-like spatial distribution of the wave intensity. Dispersion surfaces are built and special points in them are detected determining resonant frequencies. In the listed above lattices, development of resonant waves is calculated with the aim to reveal their common dynamic properties and different features depended on the cell form. Lattices possessing predetermined and randomly distributed defects are examined in order to determine the sensitivity of star-like localization forms and to discover special features inherent in such lattices.

Surprising simplicity of dislocation dynamics. Implications for thermodynamics of plasticity

Victor L. Berdichevsky

Wayne State University, USA

It was shown recently (Berdichevsky, 2017, JMPS, 106, 95-132) that 2D dynamics of edge dislocations has surprisingly simple structure: there is the only driving parameter, the dislocation polarization tensor, while all other variables are slave, i.e. in slow evolution they are functions of the driving parameter. This picture changes in avalanches, when all variables change fast. The emerging structure of dislocation phase space allows one to complete thermodynamics of plasticity which involves entropy and temperature of dislocation microstructure associated with statistical characteristics of slip avalanches (Berdichevsky, 2018, Int. J. Eng. Sci., 128, 24-30). In this talk I will show that 3D dislocation dynamics has the same structure of phase space and thus obey to the same thermodynamic laws and outline a thermodynamic theory of stress-strain curves.

Static and dynamic properties of conventional and auxetic multilattices

Igor Berinskii

Tel Aviv University, Israel

A relatively simple two-parametric model is proposed to perform a qualitative analysis of cellular auxetic materials. The model describes an interaction between the cells' nodes with axial and torsional linear springs. The homogenization procedure is proposed to determine the effective properties of solid material corresponding to the re-entrant and regular honeycombs. The procedure is based on a comparison of strain energies of the structures and corresponding orthotropic material. The components of the stiffness tensor and Poisson's ratio of the structures were studied as a function of interaction parameters and the angles between the structural elements. It was shown that re-entrant honeycombs always demonstrate the auxetic properties, whereas the isotropic regular honeycombs can have negative Poisson's ratio at some combination of axial and torsional stiffness.

The dynamic properties are studied by investigation of equations of motion for unit cell. Plane wave propagation in hexagonal and re–entrant lattices is investigated through the application of Bloch's theorem. Dispersion relations are obtained and band structures are discussed.

The dynamics of folding in Cosserat materials

Davide Bigoni[1], Panos Gourgiotis[2]
[1]University of Trento, Italy; [2]Durham University, UK

Different from Cauchy elastic materials, generalized continua, and in particular constrained Cosserat materials, can be designed to possess extreme (near a failure of ellipticity) orthotropy properties and in this way to model folding in a threedimensional solid. Following this approach, folding, which is a narrow zone of highly localized bending, spontaneously emerges as a deformation pattern occurring in a strongly anisotropic solid. How this peculiar pattern interacts with wave propagation in the time-harmonic domain is revealed through the derivation of an antiplane, infinite-body Green's function, which opens the way to integral techniques for anisotropic constrained Cosserat continua. Viewed as a perturbing agent, the Green's function shows that folding, emerging near a steadily pulsating source in the limit of failure of ellipticity, is transformed into a disturbance with wavefronts parallel to the folding itself. The results of the presented study introduce the possibility of exploiting constrained Cosserat solids for propagating waves in materials displaying origami patterns of deformation.

Financial support from the ERC advanced grant 'Instabilities and non-local multiscale modelling of materials' FP7-PEOPLE-IDEAS-ERC-2013-AdG (2014–2019) is gratefully acknowledged.

Fast cracks in brittle polymers: Structure and organization of the elementary processes at the micro/mesoscales

Daniel Bonamy[1], Alizée Dubois[1], Claudia Guerra[1], Julien Scheibert[2] and Davy Dalmas[2]

[1]Alternative Energies and Atomic Energy Commission (CEA) France; [2]École Centrale de Lyon, France

Fast crack growth is the key mechanism leading to catastrophic material failure. Linear Elastic Fracture Mechanics (LEFM) provides a predictive theoretical framework to describe its dynamics. Still, when the crack speed becomes large, a variety of instabilities are observed and LEFM stops being valid. To shed light on these mechanisms, we conducted a series of dynamic fracture experiments on acrylic glass (PMMA); fast enough cracks in such polymers, indeed, go along with the formation of microcracks which leave characteristic conical marks on the fracture surfaces. And by analyzing these patterns, it is possible to reconstruct the full space-time dynamics of the microfailure events within micrometer/nanosecond resolution. During this presentation, we will see how the collective microcrack dynamics makes emerge, on the mesoscale, a new growth dynamics for the effective crack line. We will then see how these microcracking events get organized at high speeds. Finally, we will discuss the possible links between this microcrack clustering and the micro-branching instability observed at higher speeds.

Transition waves in Rayleigh-type flexural systems

Michele Brun[1], Michael Nieves[1,2], Marta Garau[2]

[1]University of Cagliari, Italy; [2]Keele University, UK

For discrete mass-spring system there is an extensive literature describing phase transition processes starting from the approach developed in [1]. Phase transition in flexural system has been analysed more recently [2, 3, 4]. In comparison with mass-spring systems, flexural structures allow for the study of a greater range of loading scenarios and physical parameters which influence the response of a structure. They may also describe failure propagation in civil engineering systems. Here we consider the role of rotational inertia in the process of phase transition in a one-dimensional flexural system. This process is assumed to occur with a uniform speed that is driven by feeding waves carrying energy produced by an applied oscillating moment and force. We show that the problem can be reduced to a Wiener-Hopf equation via the Fourier transform. The associated solution is presented and from this we identify the dynamic behaviour of the system during the transition process. The minimum energy required to initiate the phase transition process with a given speed is determined and it is shown there exist parameter domains defined by the force and moment amplitudes where the phase transition can occur. The influence of the rotational inertia of the system on the wave radiation phenomenon connected with the phase transition is also discussed. All results are supplied with numerical illustrations confirming the analytical predictions.

Acknowledgement: M.J.N. and M.B. gratefully acknowledge the support of the EU H2020 grant MSCA-IF-2016-747334-CAT-FFLAP.

References

[1] Slepyan, L.I.: Models and Phenomena in Fracture Mechanics, Foundations of Engineering Mechanics, Springer, (2002).

[2] Brun, M., Movchan, A.B. and Slepyan, L.I.: Transition wave in a supported heavy beam, J. Mech. Phys. Solids 61, no. 10, pages 2067–2085, (2013).

[3] Brun, M., Giaccu, G.F., Movchan, A., B., and Slepyan, L. I.. Transition wave in the collapse of the San Saba Bridge. Front. Mater. 1:12, (2014). doi: 10.3389/fmats.2014.00012.

[4] Nieves, M.J., Mishuris, G.S., Slepyan, L.I.: Analysis of dynamic damage propagation in discrete beam structures, Int. J. Solids Struct. 97-98, pages 699–713, (2016).

Time-dispersive behaviour as a feature of critical-contrast media

Kirill Cherednichenko

University of Bath, UK

I will discuss a novel approach to critical-contrast homogenisation, which allows to see homogenisation limits restricted to the "stiff" component of the composite as a class of time–dispersive media. By an inversion of this argument, we also offer a recipe for the construction of such media with prescribed dispersive properties from periodic composites. This is joint work with Y. Ershova (Bath) and A. Kiselev (St.Petersburg).

Optimal Multimaterial Elastic Structures

Andrej Cherkaev
University of Utah, USA

The paper describes various structures of optimal multi–material com- posites, the range of the effective properties that are achievable by these structures, and optimal designs from two elastic materials and void. Technological capabilities, such are 3d printing and microfabrication, allow a huge variety of structures to be manufactured for roughly the same price, and one wants to know "the best" structure, or how composite microstructures can be optimized. The vast majority of obtained results deals with two–material composites because of theoretical limitations. Meanwhile, numerous applications call for the optimal design of multimaterial composites, or of porous composites made of two materials and void. The structure is called optimal (the stiffest) if it stores minimal elastic energy under a given loading. We find the lower bound that estimates the energy of any composite independently of its structure, it is constructed using the modified translation method [1, 2]. The energy of specially chosen microstructures realizes this bound. We demonstrate that the found optimal microstructures of multimaterial composites differ drastically from two-material ones. Optimal three–material structures show a large variety of patterns, and the optimal topology de- pends on the volume fractions [1, 3]. Depending of volume fraction of the mixed materials they may or may not contain a stiff envelope, they include spots of intermediate material connected by anisotropic "pathways" i.e., laminates from the strong and weak materials, "spiked wheels" assemblages [3], and other 2D and 3D-configurations that reveal a geometrical essence of optimality [4]. These structures are found with the hints provided by the modified translation bound that suggest the range of elastic fields inside each material in an optimal layout. We also demonstrate optimal designs from the found optimal composites [4]; which is the distribution of optimal composites in the designed domain.

References

[1] Cherkaev, A., 2009. Bounds for effective properties of multimaterial two-dimensional conducting composites and

elds in optimal composites. Mechanics of Materials 41, 411-433.

[2] Cherkaev, A., Zhang, Y. 2011. Optimal anisotropic three-phase conducting composites: Plane problem. International Journal of Solids and Structures 48 (20), pp. 2800-2813

[3] Cherkaev, A. 2011. Optimal Three-Material Wheel Assemblage of Conducting and Elastic Composites. International Journal of Engineering Science, Volume 59, October 2012, Pages 27-39.

[4] Andrej Cherkaev, Grzegorz Dzierzanowski. 2013. Three-phase plane composites of minimal elastic stress energy: High-porosity structures. Int. J. Solids and Structures. 50, 25?26, 2013, Pages 4145-4160

[5] Nathan Briggs, Andrej Cherkaev, Grzegorz Dzierżanowski 2017. A Note on Optimal Design of Multiphase Elastic Structures Structural and Multidisciplinary Optimization Int. J. Solids and Structures 51 (3), 749-755

Internal variables: from microscale geometry to viscosity on a large scale

Elena Cherkaev

University of Utah, USA

Formulations of problems governing the behavior of the fields in heterogeneous media often contain internal or hidden variables that reflect the influence of the processes on the microstructural scale. The talk discusses internal variables that are naturally related to the structure of a composite material. The equations for the evolution of these variables are determined by the spectral measure in the Stieltjes analytic representation of the effective properties of the composite. The spectral measure contains all information about the geometry of the media on the microscale and links it through the internal variables to the behavior of the material on the macroscale.

Geometry, topology and graded material properties and their influence upon wave propagation in structured media

Richard Craster

Imperial College London, UK

In this talk I will address topics around how the geometry of the lattice, its lattice topology, and variation of the lattice all affect the propagation of waves in phononic crystal devices. In particular wave guiding around corners, beam splitters and surface bulk wave mode converters will be presented and discussed.

Boundary layer due to inertia in the plastic zone near a rapidly propagating crack tip

Victor Dunayevsky

Shell International Exploration and Production, USA

For arbitrary values of the dimensionless crack tip speed (M) the inner solution (Slepian's solution) consists of central–fan field ahead of the crack tip and a uniform field in its wake. It is shown that this solution with a strong influence of dynamic effects is valid in a "boundary layer" which shrinks on the crack tip in the limit of vanishing M. For small crack speed the outer solution was found as a regular perturbation expansion in M with the quasi–static solution recovered as its leading term. The composite solution matching Slepian's boundary layer and logarithmically singular quasi–static solution was constructed.

On the anti-plane surface waves in media with surface energy

Victor A. Eremeyev
Gdańsk University of Technology, Poland

Surface waves play an important role in mechanics of solids and fluids. In particular, such waves may be used for manufacturing of various devices of acoustoelectronics as they may carry information on the material properties in the vicinity of the free surface. Here we discuss new type of surface anti-plane waves localized near the surface an elastic half-space considering surface strain and kinetic energies. Unlike classic linear isotropic elasticity for materials with surface energy there exist anti-plane surface waves [1, 2]. We consider the surface elastic properties starting from the discrete model of the elastic coating based on the theory of polymeric brushes. The model consists of chains of rigid rods with elastic joints. We assumed the linear elastic interaction between neighboring elements of the same chain as well as the interaction between neighboring links of different chains. The elastic energy is similar to the Stockmayer potential used one in the theory of polymers. The latter is a Lennard–Jones potential with additional term responsible for dipole interactions between neighboring elements of polymeric chains. As a result, we have in the model translational and rotational interactions between chains. Using the homogenization approach proposed in [3] we obtain the elastic parameters used in the surface elasticity. The dispersion relations are derived and their dependence on the material parameters are analyzed.

References

[1] Eremeyev, V. A., Rosi, G., Naili, S. (2016). Surface/interfacial anti-plane waves in solids with surface energy. Mechanics Research Communications, 74, 8-13.

[2] Eremeyev, V. A., Rosi, G., Naili, S. (2018). Comparison of anti-plane surface waves in strain-gradient materials and materials with surface stresses. Mathematics and Mechanics of Solids, 1081286518769960

[3] Eremeyev, V. A. (2016). On effective properties of materials at the nano-and microscales considering surface effects. Acta Mechanica, 227(1), 29-42

Kinks in chains with on-site bistable nondegenerate potential: Beyond traveling waves

Oleg Gendelman
University of Utah, USA

This talk revisits the well-known transition fronts (kinks) in chains of coupled oscillators with nondegenerate on-site potentials. Usually, such transition fronts are considered in terms of traveling-wave solutions. We explore the loss of stability of such traveling waves. Generically, it corresponds to one of the common scenarios for fixed points of discrete maps. For example, one can encounter the quasiperiodic kink propagation (due to Hopf bifurcation), or the Feigenbaum cascade of period doublings, leading to a chaotic-like propagation pattern. The aforementioned scenarios show up, for instance, for tri-parabolic and $\phi 4$ on-site potentials. Numeric evidence suggests that the loss of stability occurs due to resonances between the frequency associated with the kink propagation, and the linear band gaps of the chain. Particular resonance mechanisms are model dependent. For the classical Atkinson-Cabrera model with a bi parabolic on-site potential, the stability threshold is estimated by the simple means of linear algebra. The loss of stability in this model occurs through Hopf bifurcation. The results are in good agreement with numerical simulations.

Solitary waves in a bistable lattice

Sefi Givli, Shmuel Katz
Technion — Israel Institute of Technology, Israel

The design of architectured materials with bistable building blocks holds exciting possibilities. This new class of metamaterials exploits micro-level structural instabilities to obtain extraordinary physical and mechanical properties. Still, the dynamic behavior of these lattice structures is largely unexplored. Here, we study the dynamic response of a 1-D bistable lattice, i.e. a FPU chain with springs having a non-convex double-well energy potential. In addition to metamaterials, this model-problem is prototypical to a large number of systems, such as unfolding/refolding of proteins, crack propagation, plasticity, and mechanisms underlying martensitic phase transformations. We show that, depending only on the stiffness-ratio of the two energy wells of the bistable springs, the system exhibits two fundamentally different responses to impact; either the impact energy is (almost entirely) trapped in the form of large undulations of the first few springs, or the energy of the impact is (almost completely) transmitted along the chain in the form of a solitary wave that involves transition to the secondary energy-well and back. Focusing our attention to the latter, we reveal, based on analytical treatment and extensive numerical simulations, a universal feature of the solitary wave. Namely, the height of the solitary wave is indifferent to the energy barrier separating the two equilibria of the double well potential. This remarkable feature indicates that the spinodal region of the double–well potential affects the behavior only through its breadth.

Features of steady–state fracture in double chains

Nikolai Gorbushin[1], Gennady Mishuris[2]

[1]Université Paris-Est Créteil, France; [2]Aberystwyth University, UK

The early works of L.I. Slepyan [1,2] on dynamic fracture in linear lattices provided an efficient technique to handle problems related to crack propagation, phase transitions and shocks in discrete media. This approach enabled to link microscopic parameters, defining structural changes, to macroscopic ones. Most of the results so far concern homogeneous structures whereas there are several effects which are observed in dissimilar chains and lattices.

In this work we focus on 1d fracture problem in a double chain of different materials linked together. The crack movement is provided by concentrated constant forces. The solution derivation and its analysis is similar to [3]. We obtain explicit relations between amplitude of the forces, excitation frequency and crack velocity. Moreover, the relation between the energy release rate and crack speed is demonstrated. The theoretical study is supported by complementary numerical simulations. We show that there exist stable crack propagation at intersonic speeds with the presence of velocity gap at high speeds. Moreover, we show the peculiarity related to the choice of external loads in dynamic case. We also show that for certain choice of material properties it is possible to observe admissible steady-states in slow cracks which usually appear to be unstable.

References

[1] Slepyan, L. I., Troyankina, L. V. (1984). Fracture wave in a chain structure. Journal of Applied Mechanics and Technical Physics, 25(6), 921-927.

[2] Kulakhmetova, S. A., Saraikin, V. A., Slepyan, L. I. (1984). Plane problem of a crack in a lattice. Mech. Solids, 19(3), 102-108.

[3] Gorbushin, N., Mishuris, G. (2018). Dynamic fracture of a discrete media under moving load. International Journal of Solids and Structures, 130, 280-295.

Asymptotic formulas for the hard-sphere limit of atomic chains

Michael Herrmann

Technical University of Braunschweig, Germany

The high-energy limit of Fermi-Pasta-Ulam-Tsingou chains with Lennard-Jones-type interaction potential is naturally related to the hard-sphere model of elastically colliding particles. In this talk we study solitary waves with large speed in the chain and derive approximation formulas for the underlying advance-delay-differential equation in terms of a low-dimensional shape ODE. We also discuss the dynamical stability problem and sketch possible generalizations.

Dynamics of Structural Transformations under Mechanical Impact

Nikita. F. Morozov, Dmitry A. Indeitsev, Boris N. Semenov, Dmitry S. Vavilov
Institute for Problems in Mechanical Engineering of Russian Academy of Sciences, Russia

Numerous experiments on shock-wave loading of metals demonstrate that the crystal structure of material can be transformed in a certain range of striker velocities. At the macro-scale these transformations are observed as energy loss on the temporal velocity profile of the back side of the sample surface which contains key information about the material properties. In the present work a two-component model of material with nonlinear interaction force is proposed for description of structural transformations. Dynamic equations are written with respect to the motion of the center of mass of the components, regarded as a measured macro-parameter and with respect to their relative displacement which acts as an internal degree of freedom responsible for transition of material from one state to another. The proposed model is applied for solving a quasi-static problem of the kinematic extension of a two-component rod in order to determine the parameters of a non-monotonic stress-strain relation that is often used for describing materials subjected to phase transitions. By solving a dynamic problem the effect of non-stationary wave damping, associated with the dissipation of energy in structural changes, is demonstrated. On the basis of a continuous-discrete analogy an analytical expression is obtained for estimating the duration of structural transformations and the parameter characterizing the nonlinear interaction force. The obtained results are confirmed by a numerical solution of the nonlinear Cauchy problem using finite difference method.

Viscosity of suspensions containing particles of diverse shapes and orientations

Mark Kachanov

Tufts University, USA

We first show that the simplest, non-interaction approximation, if formulated properly has much wider applicability than it is commonly believed. This formulation involves the fluidity —- rather than viscosity —- contributions of individual particles to the overall property. It is shown that the classical Einstein's formula corresponds to an incorrect version of the non-interaction approximation and violates Hashin–Shtrikman bounds. The analysis is extended to mixtures of particles of diverse shapes (for which the volume fraction ceases to be a proper concentration parameter) and to cases of anisotropic viscosity.

Multiscale modelling of materials chemomechanics: from stress corrosion cracking to catastrophic brittle fracture

James Kermode
University of Warwick, UK

Fracture is the dominant failure process underlying many materials reliability issues. At the same time, it remains one of the most challenging "multi-scale" modelling problems, requiring both an accurate description of the chemical processes occurring in the near tip region and the inclusion of a much larger region in the model systems. I will explain how these requirements can be met simultaneously by combining a quantum mechanical description of the crack tip with a classical atomistic model that captures the long-range elastic behaviour of the surrounding crystal matrix, using a QM/MM (quantum mechanics/molecular mechanics) approach such as the 'Learn on the Fly' (LOTF) scheme [1]. The talk will be illustrated with a range of examples in covalent, oxide and metallic systems [2-8].

References

[1] G. Csányi et al., Phys. Rev. Lett. 93, 175503 (2004)
[2] Z. Li, J. R. Kermode and A. De Vita, Phys. Rev. Lett. 114, 096405 (2015)
[3] F. Bianchini et al., Modell. Simul. Mater. Sci. Eng. 24, 045012 (2016)
[4] E. Bitzek, J. R. Kermode & P. Gumbsch Int. J. Fract. 191, 13–30 (2015).
[5] J. R. Kermode et al., Nature 455, 1224-1227 (2008)
[6] J. R. Kermode, et al., Nat. Commun. 4, 2441 (2013)
[7] J. R. Kermode et al., Phys. Rev. Lett. 115, 135501 (2015)
[8] A. Gleizer et al., Phys. Rev. Lett. 112, 115501 (2014)

Arrays of coupled micro cantilevers with varying length

Christopher B. Wallin[1,2], Roberto De Alba[1,2], Daron A. Westly[1], Scott Grutzik[3], Nir Dick[6], Alan T. Zehnnder[4], Richard H. Rand[4,5], Vladimir Aksyuk[1], B. Robert Ilic[1], Slava Krylov[6]

[1]National Institute of Standards and Technology, USA; [2]University of Maryland, USA; [3]Sandia National Laboratories, USA; [4]Sandia National Laboratories, USA; [5]Cornell University, USA; [6]Tel Aviv University, Israel

Large arrays of micro- and nanoelectromechanical (MEMS/NEMS) coupled nonlinear resonators have been shown to possess complex dynamics such as parametric resonances, intrinsically localized modes, abrupt transitions between standing wave patterns, and sensitivity to environmental parameters. Here we explore, both numerically and experimentally, the collective dynamics of large arrays of interdigitated micro cantilevers interacting elastically and electrostatically, through flinging electrostatic fields. Finite element modal analysis is implemented to determine the spectral behavior of the array and to extract a fully populated coupling matrix. A compact reduced order Galerkin model is used for numerical investigation of the array's dynamic response. Arrays of various geometries including interdigitated cantilevers of nominally identical and linearly varying length were fabricated using a silicon on insulator (SOI) wafer with a highly doped single crystal silicon device layer. The devices were driven electrically and inertially, using a piezoelectric transducer, under ambient and vacuum conditions. Our results show that at a given excitation frequency within a propagation band, only a finite number of beams respond. Spectral characteristics of individual array elements, inertially excited by an external piezoelectric actuator, were measured using laser interferometry. The theoretical and experimental results collectively show that the resonant peaks corresponding to individual beams are clearly separated when operating in vacuum at the third harmonic. Distinct resonant peak separation coupled with the spatially confined modal response makes higher harmonic operation of tailored, variable length cantilever arrays well suited for a variety of resonant based sensing applications. Electrostatic actuation was explored as well. In this case interactions between interdigitated cantilevers took place through fringing electrostatic fields within the overlap region and can be tailored by varying the actuation voltage. Our experimental and model results show that high vibrational amplitudes and abrupt transitions between the standing wave patterns can be achieved by excitation through the parametric resonance mechanism.

References

[1] S. Krylov, S. Lulinsky, B. R. Ilic, I. Schneider, "Collective Dynamics and Pattern Switching in an Array of Parametrically Excited Micro Cantilevers Interacting Through Fringing Electrostatic Fields," Applied Physics Letters, 105, 071909, 2014.

[2] N. Dick, S. Grutzik, C. B. Wallin, B. R. Ilic, S. Krylov, A. T. Zehnder, "Actuation of Higher Harmonics in Large Arrays of Micromechanical Cantilevers for Expanded Resonant Peak Separation," J. of Vibration and Acoustics, 140, pp. 051013-1 - 051013-10, 2018.

Matrix factorization in problems for cracks in elastic media

Pavlos Livasov, Gennady Mishuris
Aberystwyth University, UK

There is a number of techniques, like complex function theory and integral transforms, used in the solution of problems of fracture mechanics which can be reduced to Wiener– Hopf functional equations defined in a strip in a complex transform plane [1]. This applies to both static problems within a continuous model [2] and dynamic problems, especially when it comes to steady-state regime. This technique is also effective in the case of discrete problems [3,4], which concern both lattice structures composed of masses and connecting springs [5] and structures made of masses and beams [6]. Wiener–Hopf technique allows one to determine the basic properties of the solution and to identify important physical applications relating to the nature of crack propagation or phase transitions [7]. In some cases when there is a significant process zone in the vicinity of the crack tip, the application of this method yields coupled systems of equations, and the resulting Wiener–Hopf kernels are of matrix form. Apart from some special cases, the solution is obtained by means of approximate methods [8,9]. In the present paper, some of these problems are considered and respective numerical examples are presented and discussed.

Singularities of dynamic cracks

Michael Marder
University of Texas at Austin, USA

The continuum theory of fracture limits cracks in Mode 1 loading to the Rayleigh wave speed. Yet experiment and numerical find no such limit so long as the crack tip remains stable. I will explain how the singularity near a crack tip changes as it approaches and passes the Rayleigh wave speed and how to resolve the apparent paradox.

Crack dynamics in a bimaterial lattice

Nikolai Gorbushin[1], Gennady Mishuris[1],Andrea Piccolroaz[2]
[1]Aberystwyth University, UK;[2]Universit di Trento, Italy

We consider an infinite rectangular bimaterial lattice with interface links having different properties than those of the adjacent structures. A semi-infinite crack propagates along the interface generating waves. We construct the dispersion diagrams, analysing possible waves propagating in the biomaterial structure. The propagation of the crack is induced by an external load generating feeding waves which bring energy to the crack front bonds and cause their disintegration. In turn, this produces dissipative waves which carry energy away from the crack front. Using the approach of Slepyan [1], an equation of the Wiener-Hopf type is derived and solved along the crack face. The crack stability is analysed via the evaluation of the energy release rate for different contrasts in the properties of the glued structures. Using this model, we present numerical illustrations showing the effect of mismatch in properties of lattices and also the effect of the introduced interface. Admissible and forbidden regimes are identified on the basis of the assumed fracture criterion [2].

Acknowledgments. A.P. and G.M. gratefully acknowledge financial support from ERC-2013-ADG-3405- 61-INSTABILITIES. N.G. gratefully acknowledges financial support from the European Unions Seventh Framework Programme FP7/2007-2013/ under REA grant agreement number PITN-GA-2013-606878-CERMAT2. G.M. is also thankful to the Royal Society for the Wolfson Research Merit Award.

References

[1] Slepyan, L.I. Models and Phenomena in Fracture Mechanics, (2002) Springer, 557 pages.

[2] Gorbushin, N., Mishuris, G. On admissible steady-state regimes of crack propagation in a square-cell lattice. arXiv:1701.04818 (2017).

Negative refraction in quasicrystalline multilayered metamaterials

Lorenzo Morini, Y. Eyzat, Massimiliano Gei
Cardiff University,UK

We investigate the problem of an antiplane wave obliquely incident at the interface between an elastic substrate and a laminate is investigated. The considered layered media possess a quasicrystalline structure, generated according to the Fibonacci substitution rules [1]. The substrate-laminate system is studied combining the transfer matrix method to the normal mode decomposition technique [2]. The diffraction angles associated with the transmitted modes are estimated by means of the space averaging procedure of the Poynting vector [3]. We show that, with respect to a periodic classical bilayer [4], on the one hand, beyond a certain frequency threshold, high order Fibonacci laminates can provide negative refraction for a wider range of angles of incidence, on the other, they allow negative wave refraction at lower frequencies. Moreover, the performed numerical results illustrate that the Bloch-Floquet spectrum corresponding to this class of laminates has a self-similar character linked to the specialisation of the Kohmoto's invariant, a function of the frequency that was recently studied by the authors for periodic one-dimensional quasicrystalline-generated waveguides [5]. This function is able to explain two types of scaling occurring in dispersion diagrams. The obtained results represent an important advancement towards the realisation of multilayered quasicrystalline metamaterials.

References

[1] Gei M. Waves in quasiperiodic structures: stop/pass band distribution and prestress effects, Int. J. Solids Struct., 47 (2010) 3067-3075.

[2] Srivastava A. and Willis J.R. Evanescent wave boundary layers in metamaterials and sidestepping them through a variational approach, Proc. R. Soc. A, 473 (2017) 20160765.

[3] Willis J.R. Negative refraction in a laminate, J. Mech. Phys. Solids, 97 (2016) 10-18.

[4] Srivastava A. Metamaterial properties of periodic laminates, J. Mech. Phys. Solids, 96 (2016) 252- 263.

[5] Morini L. and Gei M. Waves in one-dimensional quasicrystalline structures: dynamical trace mapping, scaling and self-similarity of the spectrum, J. Mech. Phys. Solids, 119 (2018) 83-103.

Dynamics and stability of axially loaded flexible rod

Nikita F. Morozov[1,2], Petr E. Tovstik[1], Tatiana P. Tovstik[1,2], Alexander K. Belyaev[2], Chien-Ching Ma[3]

[1] St.Petersburg State University, Russia; [2]Institute for Problems in Mechanical Engineering of Russian Academy of Sciences, Russia; [3]National Taiwan University, Taiwan

The presentation addresses dynamic bucking of thin rod subjected to discontinuous constant longitudinal load. The transverse deflection of the rod is sought for as a Fourier series in terms of the orthonormal system of normal modes of bending vibration. The problem is solved in both linear and nonlinear approximations. The evolution of the beam deflection is displayed. A number of effects are found out. Among these, it is shown that the dynamic buckling can occur if the magnitude of the axial load is less than the Euler critical force, however the effect is caused by a parametric resonance. The analytic work is confirmed by dynamic simulation in the framework of geometrically nonlinear solid mechanics. A benchmark study of the case of axial impact is carried out by means of Sears Approach, element analysis and experiment.

From motorcycles to elastic chiral metamaterials: localisation and wave guiding

Alexander B. Movchan
University of Liverpool, UK

The main component of a motorcycle is a front wheel (not all would agree though). If considered as a gyroscope it provides an insight into a counterintuitive way to steer a motorcycle at a high speed– countersteering.

A periodic arrangement of gyroscopes connected by elastic links may give an unusual structure whose dynamic response may appear counterintuitive. Directional preference imposed by the direction of spin of gyroscopes makes the structure chiral, and the dispersion relations for elastic waves In such systems are unusual. Analytical model is presented for a chiral elastic periodic multi-structure. Dispersion , dynamic anisotropy, localisation and wave guiding are shown for a range of different frequencies. Theoretical findings are accompanied by illustrative examples.

The lecture is based on the results of the joint work with N.V. Movchan, I.S. Jones, M. Brun, G. Carta and M.J. Nieves.

Dynamics of elastic chiral multi-structures

Michael J. Nieves[1,2], Giorgio Carta[3], Ian S. Jones[3], Alexander B. Movchan[4] and Natalia V. Movchan[4]

[1]Keele University, UK; [2]University of Cagliari, Italy; [3]Liverpool John Moores University, UK; [4]University of Liverpool, UK

We discuss the interaction of an Euler-Bernoulli beam with a gyroscopic spinner, which imparts chirality to the elastic system [1,2]. Effective boundary conditions characterising the interaction between the beam and the spinner are derived and the dynamic properties of this chiral system are analysed in detail. The conditions are then adapted to analyse waves propagating in systems composed of periodically placed gyroscopes connected by beams. In the low frequency regime, we demonstrate this system behaves as a gyro-elastic medium [3], known as the Rayleigh gyrobeam. The results presented are accompanied by illustrative numerical examples. Potential applications are foreseen in aerospace engineering for the control of space flight [4] and in civil engineering for the design of novel earthquake protection systems [5].

References

[1] G. Carta, M.J. Nieves, I.S. Jones, N.V. Movchan and A.B. Movchan (2018): *Elastic chiral waveguides with gyro-hinges*, Quart. J. Mech. Appl. Math. 71, pp. 157-185.

[2] M.J. Nieves, G. Carta, I.S. Jones, A.B. Movchan and N.V. Movchan (2018): *Vibrations and elastic waves in chiral multi–structures*, (submitted).

[3] G.M.T. D'Eleuterio and P.C. Hughes (1984): *Dynamics of gyroelastic continua*. J. Appl. Mech. 51, 415–422.

[4] G.M.T. D'Eleuterio and P.C. Hughes (1987): *Dynamics of gyroelastic spacecraft*. J. Guidance 10, no. 4, 401–405.

[5] G. Carta, I.S. Jones, N.V. Movchan, A.B. Movchan and M.J. Nieves (2017): *Gyro-elastic beams for the vibration reduction of long flexural systems*, Proc. Math. Phys. Eng. Sci. 473, 20170136.

The suitability of the effective wavefield as a tool to predict wave attenuation over long distances

Malte Peter

Augsburg University, Germany

For an incident wave train propagating into a rough (randomly disordered) medium, wave localisation refers to exponential attenuation (on average) of the wave train in the rough medium. A key quantity of interest is the attenuation rate as a function of the incident wave properties (frequency) and the properties of the given medium, including the statistical properties of the disorder in media. Effective media theory is an appealing way to approach the problem, as it provides analytical insight, circumventing the need to compute individual wave fields repeatedly for different realisations of the disorder, as well providing the opportunity for elegant mathematical analysis. I will present the theory alongside corresponding results and discuss the applicability of effective media theory for making predictions of attenuation of (linear) waves over long distances in media with continuously varying properties.

This is joint work with L. G. Bennetts (Adelaide) and S. Rupprecht (Augsburg).

Wallner lines: A new look at an old problem

Krishnaswamy Ravi-Chandar
University of Texas at Austin, USA

Wallner identified correlated pattern formation in dynamically growing crack surfaces about 80 years ago; analysis of the geometry of this has been commonly used for determination of crack speeds in the absence of direct measurements. In recent years, an alternate model, based on the idea of a new type of characteristic wave called the crack front wave, associated with perturbations generated along crack fronts that persist for long time and propagate along the crack front, has been presented. We examine the surface patterns on dynamically fractured surfaces at high spatial resolution, and obtain clues to their formation, and attempt to reconcile between the two models: specifically, we show that these patterns are closely tied to the problem of initiation and growth of cracks under combined mixed-mode I+III loading. The presentation will begin with an overview of Wallner lines, takes detour to mixed mode I+III fracture, and finish with inferences on Wallner line formation.

Instabilities in advancing hydraulic fracture fronts

Shmuel M. Rubinstein
Harvard University, USA

Fracture surface roughness can play a significant role in enhanced oil recovery, due to its impact on the fluid dynamics within the thin fracture. In theory, fracture surfaces should be smooth; however, undulations and crinkles of the fracture front resulting from heterogeneities and dynamic instability produce a complex fracture surface. These effects are fast, multi scale and generically three dimensional, and as such, are intractable both experimentally and computationally. Experimentally, these difficulties are mediated by studying fracture dynamics in brittle hydrogels. In these transparent materials, the fracture dynamics are slow and can be visualized. We combine high speed photography and laser sheet microscopy and directly observe in full 3D how roughness is dynamically generated by the fracture front. Specifically, I will discuss the behavior of small step–like discontinuities that form, propagate, and interact with each other along the advancing front. The interaction of these step lines can create significant relief along the fracture front and can even result in fracture propagation on separate parallel planes.

Evenly spread damage in the fault-tolerant bi-lattice

Michael Ryvkin[1], Andrej Cherkaev[2], Stephan Rudykh[3], Viacheslav Slesarenko[4]
[1]Tel Aviv University, Israel;[2]Univesity of Utah, USA;[3]University of Wisconsin Madison, USA; [4]Technion, Haifa, Israel

A usual failure mode for homogeneous brittle solid material subjected to a tensile loading is a catastrophic crack propagation, which is accompanied by a strain localization. By contrast, failure in a meta-material with fault-tolerant microstructure is graduate: the appearance of a crack is preceded by a preliminary stage when partly damaged material just reduced its stiffness but does not lose its structural integrity, the damage is not localized and it is distributed evenly in the material volume. As a result, it is characterized by a high value of absorbed energy per unit mass.

A design of fault-tolerant two-dimensional lattice material is presented. First damage propagation in homogeneous triangular stretch-dominated and hexagonal bending-dominated lattices for different failure criteria is considered [1]. It is modeled as a sequential failure of beam elements. The considered design object is a heterogeneous isotropic lattice(bi-lattice) can be viewed as a combination of these two lattices. Bi-lattice is composed of beam elements with two different thicknesses [2]. Periodic redistribution of the material between the beams for the fixed lattice density leads to the optimal layout for which only thin (sacrificial) links failed at the first stage of damage propagation. Stronger links in between of the weak elements arrest a single flaw growth, and damage develops as a cloud of separated small flaws. A macrocrack in such lattice appears only after failure of all distributed sacrificial links, this property significantly increases the energy absorbing rate.

The theoretically obtained fault-tolerant design was confirmed by experiment. In the uniaxial tension test on bi-lattice made of elasto-plastic material several non-localized periodic deformation modes were observed. At the first stage, some of the separated sacrificial elements elastically buckled, at the second stage yielding takes place for another group of the elements, which followed by their rupture.

References

[1] Cherkaev, A. and Ryvkin, M. Damage propagation in 2d beam lattices: 1. Uncertainty and assumptions, *to appear in Archive of Applied Mechanics.*

[2] Cherkaev, A. and Ryvkin, M. Damage propagation in 2d beam lattices: 2. Design of an isotropic fault-tolerant lattice, *to appear in Archive of Applied Mechanics.*

Replacement relations in micromechanics

Igor Sevostianov
New Mexico State University, USA

Usually, replacement relations link overall properties of heterogeneous materials that have the same matrix material and microstructure while properties of the inclusions are different. First relations of this kind have been proposed by Gassmann (1951) in the context of the effect of saturation on seismic properties of rock in geomechanics. He proposed to express the bulk modulus of fully saturated rock in terms of the elastic properties of dry rock. His results were further developed in the works of Brown and Korringa (1975) and Saxena and Mavko (2014). In the context of thermal conductivity problem, similar results have been obtained by Keller (1964) and Schulgasser (1976) in 2-D and by Zimmerman (1989) in 3-D. In this presentation, we discuss replacement relations for elastic properties and conductivity adopting approach based on property contribution tensors. We show that the replacement relations can be applied with good accuracy for materials containing non-ellipsoidal inhomogeneities of convex shape while the error is significant for concave shapes. For this case, a modification involving an extra shape factor can be proposed. Also we derive replacement relations for conductivity of the heterogeneous materials that have different matrices. The most important role of the relation of this kind is that it allows evaluation of the thermal conductivity of inhomogeneities via measurement of the overall thermal conductivities of two composites with two different known matrices. The result is independent of the shape of the inhomogeneities. The theoretical derivations show good agreement with experimental measurements.

References

Gassmann F. Über die elastizität porpöser medien, Vierteljahrsschrift der Naturforschenden Gesellscaft in Zurich, 1951; 96: 1-23.

Brown RJS and Korringa J. On the dependence of the elastic properties of a porous rock on the compressibility of the pore fluid. Geophysics, 1975; 40: 608–616.

Keller, J.B. A Theorem on the conductivity of a composite medium. Journal of Mathematical Physics, 1964; 5: 548-549

Saxena N and Mavko G. Impact of change in pore-fill material on P-wave velocity, Geophysics, 2014; 79: D399-D407.

Schulgasser, K. On a phase interchange relationship for composite materials; Journal of Mathematical Physics, 1976; 17: 378-381. Zimmerman RW. Thermal conductivity of fluid-saturated rocks, J. Pet. Sci. Eng., 1989; 3: 219–227.

On some examples of discrete scattering in simple structures and their applications in mechanics and physics

Basant Lal Sharma

Indian Institute of Technology in Kanpur, India

In the presence of various length scales, the wave propagation problem, as well as those involving other kinds of dynamical phenomena, starts to reveal new effects due to interplay between wave dispersion and structural features. Some of these effects have been also used to model and synthesise new kinds of materials by various researchers around the world. The underlying mechanics, however, in itself, is an enchanting arena to explore. Indeed, amidst these developments, the theory of discrete scattering has experienced a kind of revival during the last two decades, partly due to the rising importance of nanoscale structures in scientific and technological applications. The present talk concerns an analysis of discrete scattering effects in certain simple structures.

In a series of recent papers, so far, the mathematical framework has been developed for only scalar waves; such as anti-plane shear waves for elastic media. The elementary lattice models that have been investigated involve application of standard techniques in mathematical analysis. Besides the two dimensional 'infinite' media, waveguides are also interesting from the scattering viewpoint. In classical waveguides such as plates, rods, cylindrical shells, layered solids, etc., the 'neighbouring' parallel boundaries 'guide' the waves. These are well established subjects in mathematical sciences; for example, the research works on elastic waveguides can be traced to 1876, around the well-known contributions due to Rayleigh and Lamb. The discrete counterparts of two dimensional media and quasi-one dimensional waveguides, i.e., lattices and lattice waveguides, respectively, also find many physical applications, such as pure crystals dynamics in harmonic approximation and also in the presence of impurities, vibrations of macro-molecules, chemical applications. The technological interests, some of them quite recent, involve phonon, electronic, photonic, mechanical, as well as magnetic-spin related transport in nanostructures; occasionally these apparently diverse applications involve the governing equations, which have similar mathematical structure. A couple of these applications have been also found relevant for certain tractable problems discussed in this talk.

As a prototype example of diffraction phenomenon, which is frequently present in above mentioned context, a discrete analogue of Sommerfeld diffraction by a half plane has been recently analysed for three different lattice structures: square, triangular, and hexagonal. Their are other classical canonical problems in diffraction theory also which can be tackled from the same lattice viewpoint; for example bifurcated waveguides. The two simple cases that have been recently studied correspond to semi-infinite row with discrete Neumann condition and discrete Dirichlet condition. Both kinds of half-row 'defects' have been analyzed in infinite lattices as well as in waveguides using Fourier analysis. Exact solutions as well as their approximate expressions have been constructed for the near-tip field as well as far-field. At the technical level the triangular and (zigzag) hexagonal lattices are analytically more challenging to work with as there are some difficulties in the setting up the coordinates. The talk will give a flavour of the mathematical formulation and attempt to layout the mathematical techniques which are useful to derive some physically relevant entities; for the infinite lattice, an example of the latter is far-field approximation and for the waveguides, the reflectance and transmittance. Towards the end, the talk will also include an announcement of open problems associated with these newly emerging developments as well as some reflections on the ongoing work.

The Buckling-Fracture Transition in Non-Euclidean Plates

Eran Sharon
Hebrew University of Jerusalem, Israel

Non-Euclidean Plates (NEP) are thin elastic plates, in which lateral equilibrium distances of the material are described by a non-Euclidean reference metric. When flat, such materials are residually stressed. Previous studies showed that such plates buckle spontaneously – while free of external constraints. In the thin limit the geometry of the buckled configurations approaches the reference metric. In this talk we describe experiments that show the existence of a new, buckling to fracture, transition in such plates. Depending on geometrical and mechanical parameters of the system, NEP undergo fracture instability instead, or together with, buckling instability. In such cases, fracture onset can occur in strains that are much higher than the Griffith criterion in flat plates. We propose the scaling of this transition and verify it experimentally. Our observations lead us to propose an intrinsic geometrical description of fracture, which is consistent with, but different from Linear Elastic Fracture Mechanics.

Macro to micro in fracture: From dynamic cleavage energy to atomistic bond breaking mechanisms

Dov Sherman

Tel-Aviv University, Israel

The effect of atomistic scale events on the macroscopic fracture properties of brittle crystals, the dynamic cleavage energy in particular, is not fully understood and still debated. The effect of the gradient of the energy release rate (ERR), dG_0/da, on both the atomistic bond breaking mechanisms and sequence and the cleavage energy is even less known.

We have recently completed an experimental investigation of cracks dynamics in quasi-statically loaded specimens made of brittle silicon crystal. The experiments were performed using our coefficient of thermal expansion mismatch (CTEM) method. Most importantly, the cleavage energies were evaluated for a range of driving forces, Θ. The experimental energy-speed relationship were compared with Freund equation of motion to extract the varying cleavage energies at initiation and during propagation, denoted here Γ_0 and Γ_{DM}, respectively. We show, at first, that Freund equation of motion well predicts the energy-speed relationship of dynamic cracks. We then show that both Γ_0 and Γ_{DM} are Θ dependent. Surprisingly, both are independent of crack speed.

An important macroscopic physical occurrence in our specimens is a shallow curvature of the crack front. The only topological explanation to the curvature is that it constructed from atomistic scale planar step or kinks. We estimated that hundreds of thousands of kinks exist in 680 μm thick specimen. The kinks propagate by two major mechanisms; kink advance and kink formation, with distinct energy of propagation.

We will show that the macroscopic crack front curvature and cleavage energy and the microscopic bond breaking mechanisms and sequence in the atomistic scale depend on the ratio between the number of kink advance to kink formation mechanisms; all are governed by the macroscopic Θ. While the cleavage energy is the the theoretical Griffith barrier of $2\gamma_s$ (γ_s being the free surface energy) for $\Theta < 0.5J/m^2/mm$, it significantly increase when $\Theta \approx 0.5J/m^2/mm$, indicating on the ‘ lattice trapping ’ energy, where the crack front is straight.

Geometrically Frustrated Mechanical Metamaterials

Yair Shokef
Tel Aviv University, Israel

Mechanical metamaterials are built of repeating macroscopic unit cells, and are designed so that their cooperative local deformations will lead to unusual mechanical behavior at the system level. We focus on two- and three-dimensional structures with anisotropic unit cells. When the orientation of each one is set at random they typically form an aperiodic structure, in which adjacent unit cells may not all deform self-consistently, thus constituting a mechanical spin glass. By mapping to discrete spin models, we present a combinatorial strategy for the design of a multitude of aperiodic, yet frustration-free metamaterials that exhibit spatially textured functionalities. We demonstrate these by designing three-dimensional metacubes, which when compressed can deform to give any pre-defined texture on their faces. Moreover, we quantitatively explain how pressing on a metacube with the wrong texture increases its overall rigidity. Finally, in two-dimensions we construct topological defects which give rise to multistability reminiscent of multiple metastable states in glasses.

Effect of cortical-bone microstructure on crack propagation under dynamic loading

Vadim V. Silberschmidt

Tel Aviv University, Israel

Structural integrity of bone tissue plays an important role in daily activities of humans. However, traumatic incidents such as sports injuries, collisions and falls can cause bone fracture, severe pain and mobility loss. In addition, ageing and degenerative bone diseases such as osteoporosis can increase the risk of fracture. As a composite-like material, a cortical bone tissue can tolerate moderate fracture/cracks without complete failure. The key to this is its heterogeneously distributed microstructural constituents providing both intrinsic and extrinsic toughening mechanisms. At micro-scale level, cortical bone can be considered as a four-phase composite material consisting of osteons, Haversian canals, cement lines and interstitial matrix. These microstructural constituents can directly affect local distributions of stresses and strains, and, hence, crack initiation and propagation. Therefore, understanding the effect of micromorphology of cortical bone on crack initiation and propagation, especially under dynamic loading regimes is of great importance for evaluation of fracture risks. In this study, random microstructures of a cortical bone tissue were modelled with finite elements for four groups: young (control), senior, osteoporosis and bisphosphonate-treated, based on osteonal morphometric parameters measured from microscopic images for these groups. The developed models were loaded under the same dynamic loading conditions, representing a direct impact incident, resulting in progressive crack propagation. An extended finite-element method (X-FEM) was implemented to realize solution-dependent crack propagation within the microstructured cortical bone tissues. The obtained simulation results demonstrate significant differences due to micromorphology of cortical bone, in terms of crack-propagation characteristics for different groups, with the young group showing highest fracture resistance and the senior group the lowest.

Elastic media with structures: continuous versus discrete models, modified wave equations and structure effects

Leonid I. Slepyan[1], Gennady S. Mishuris[2], Alexander B. Movchan[3]
[1]Tel Aviv University, Israel; [2]Aberystwyth University, UK; [3]University of Liverpool, UK

We consider in terms of Green's functions, macro–to–micro elastically connected structures, both continuous or discrete or one continuous and another discrete. This general representation allows us to show how an effective mass becomes negative (which corresponds to a band gap) and how the energy of a harmonic wave is transferred (partially or fully) from the waveguide to the microstructure.

We had found that the partial group velocity, as the energy flux velocity in the macrostructure, reaches a maximum when the averaged kinetic and potential energies coincide. Since such equipartition does not hold for the waveguide affected by the microstructure, the partial group velocity appears below that for a free waveguide; however, it is higher than for the composite and becoming unboundedly higher as the frequency is approaching the band gap.

Specifically, we consider the microstructure as a set of uniformly (continuously and discretely) distributed oscillators attached to or embedded into a homogeneous medium or a structure. We have generalized the Navier-Lamé and Sobolev 3D dynamic equations on an elastic medium with the microstructure. Harmonic waves in a string and a bending beam with the microstructure and localized waves in a membrane and flexible plate with line-distributed oscillators are discussed in detail.

An application of the bi-orthogonality relations in the waveguide theory

Sergey Sorokin
Aalborg University, Denmark

The reciprocity and orthogonality relations are generally recognized as robust and convenient tools for solving a broad range of forced/free wave propagation and vibration problems in elasto–dynamics, acoustics and structural mechanics. Quite surprisingly, equally, if not more powerful bi-orthogonality relations remain relatively obscure. Their application so far has been restricted to solution of the canonical Rayleigh-Lamb problem of forced response of a semi-infinite elastic layer.

The purpose of the talk is to promote the bi-orthogonality relations as the means not only to solve in a surprisingly simple way a much broader range of problems in linear dynamics of multi-modal waveguides, but also to gain an insight into formation of the eigenfrequency spectra of their segments. Furthermore, these relations dramatically simplify boundary integral equations for arbitrarily complex multi-modal waveguides and facilitate analysis of the periodicity-induced wave attenuation effects, in particular, in micro-structured materials.

The usefulness of the bi-orthogonality relations will be illustrated in several examples of progressive complexity, ranging from elementary 1D to advanced 3D composite waveguides.

Rayleigh-type waves on a coated elastic half-space with a clamped surface

Julius Kaplunov, Danila Prikazchikov, Leya Sultanova
Keele University, UK

It is well known since long ago [1] that the Rayleigh wave on a homogeneous elastic half-space only exists for a traction free surface. Recent rigorous analysis of a layered half-space with a clamped surface in [2] demonstrates a possibility of a Rayleigh-type wave for certain setups. In present study we are aiming at a multiparametric treatment of the particular problem of a coated elastic half-space with a clamped surface. For relatively small thickness and stiffness of the coating, we first obtain effective boundary conditions on the surface of a homogeneous half-space, similarly to [3]. Then, the problem is reduced to a singularly perturbed hyperbolic equation for a Rayleigh-type wave using earlier established asymptotic technique, e.g. see [3, 4]. As a result, we derive an explicit correction to the classical Rayleigh wave speed. Also, we evaluate the range of problem parameters for which the sought for surface wave exists.

References

[1] L. Rayleigh. On waves propagated along the plane surface of an elastic solid. Proc. Lond. Math. Soc., 1(1), 4-11 (1885).

[2] K.D. Cherednichenko, S. Cooper. On the existence of high-frequency boundary resonances in layered elastic media. Proc. R. Soc. A, 471(2178), 20140878 (2015).

[3] H.-H. Dai, J. Kaplunov, D.A. Prikazchikov. A long-wave model for the surface elastic wave in a coated half-space. Proc. R. Soc. A, 466(2122), 3097-3116 (2010).

[4] J. Kaplunov, D.A. Prikazchikov. Asymptotic theory for Rayleigh and Rayleigh-type waves. Adv. Appl. Mech., 50, 1-106 (2017).

Classical shear cracks drive the onset of frictional motion

Ilya Svetlizky[1], Elsa Bayart[1], David Kammer[2] and Jay Fineberg[1]
[1]Hebrew University of Jerusalem, Israel; [2]Cornell University,USA

Non-Euclidean Plates (NEP) are thin elastic plates, in which lateral equilibrium distances of the material are described by a non-Euclidean reference metric. When flat, such materials are residually stressed. Previous studies showed that such plates buckle spontaneously – while free of external constraints. In the thin limit the geometry of the buckled configurations approaches the reference metric. In this talk we describe experiments that show the existence of a new, buckling to fracture, transition in such plates. Depending on geometrical and mechanical parameters of the system, NEP undergo fracture instability instead, or together with, buckling instability. In such cases, fracture onset can occur in strains that are much higher than the Griffith criterion in flat plates. We propose the scaling of this transition and verify it experimentally. Our observations lead us to propose an intrinsic geometrical description of fracture, which is consistent with, but different from Linear Elastic Fracture Mechanics.

From Spinodal to Critical Fracture

Lev Truskinovsky
École Polytechnique, France

Fracture in disordered solids is known to be intermittent across a vast range of scales from earthquakes to micro–peeling. The observed power law avalanche has been previously linked to either spinodal or critical points. We use the simplest model to show that both associations are relevant. The realization of a particular scenario in a quasi-statically driven system with over–damped athermal dynamics depends on two parameters representing disorder and rigidity.

Strictly supersonic solitary waves in lattices

Anna Vainchtein
University of Pittsburgh, USA

We consider a nonlinear mass-spring chain with first and second-neighbor interactions and show that there is a parameter range where solitary waves in this system are strictly supersonic. In these regimes standard quasicontinuum theories, targeting long-wave limits of lattice models, are not adequate since even weak strictly supersonic solitary waves are of envelope type and crucially involve a microscopic scale in addition to the mesoscopic scale of the envelope. To capture this effect in a continuum setting it is necessary to employ unconventional, higher-order quasicontinuum approximations carrying more than one length scale. This talk is based on recent joint work with Lev Truskinovsky (ESPCI).

Material sink approach to fracture modeling

Konstantin Volokh

Technion — Israel Institute of Technology, Israel

Bulk cracks are created by massive breakage of molecular or atomic bonds. The latter, in its turn, leads to the highly localized loss of material, which is the reason why even closed cracks are visible by a naked eye. Thus, fracture can be interpreted as the local material sink. Mass conservation is violated locally in the area of material failure. We consider a theoretical formulation of the coupled mass and momenta balance equations for a description of fracture [1, 2]. Our focus is on brittle fracture and we propose a finite strain hyperelastic thermodynamic framework for the coupled mass-flow-elastic boundary value problem. The attractiveness of the proposed framework as compared to the traditional continuum damage theories is that no internal parameters (like damage variables, phase fields etc.) are used while the regularization of the failure localization is provided by the physically sound law of mass balance.

References

[1] Volokh KY (2017) Fracture as a material sink. Materials Theory 1:3

[2] Faye A, Lev Y, Volokh KY (2018) The effect of local inertia around the crack tip in dynamic fracture, submitted

From composites to metamaterials: an account of a personal journey

John Willis
University of Cambridge, UK

At Leonid Slepyan's suggestion, I shall describe how I stumbled into research in acoustic metamaterials and outline such results as I have achieved, from the distant past up to the near present.

Author Index