INdAM Workshop on

Trends on Applications of Mathematics to Mechanics

Rome, September 5-9, 2016

- **Monday, September 5, 2016**

  14:00 – 14:30 *registration*

  14:30 – 15:05 **Mark Peletier:** *Problems in the upscaling of dislocations*

  15:10 – 15:45 **Giuseppe Toscani:** *Kinetic theory of wealth distribution*

  15:50 – 16:20 *break*

  16:20 – 16:55 **Barbara Niethammer:** *Instabilities and oscillations in coagulation equations*

  17:00 – 17:35 **Arghir Zarnescu:** *Anisotropic spatial variations in liquid crystal models*

- **Tuesday, September 6, 2016**

  09:30 – 10:05 **Pablo Pedregal:** *What does rank-one convexity have to do with viscosity solutions?*

  10:10 – 10:45 **Pierluigi Colli:** *Solutions for a model related to solid-solid phase transitions in shape memory alloys*

  10:50 – 11:25 **Alain Goriely:** *Geometric stability for one-dimensional variational problems*

  11:30 – 12:00 *break*

  12:00 – 12:35 **Tommaso Ruggeri:** *Non-linear maximum entropy principle for a polyatomic gas subject to the dynamic pressure*

  12:40 – 13:15 **Marita Thomas:** *Rate-independent evolution of sets*

  13:20 – 14:30 *lunch*

  14:30 – 15:05 **Antonin Chambolle:** *Crystalline mean curvature flow*

  15:10 – 15:45 **Riccarda Rossi:** *Balanced Viscosity solutions for multi-rate systems*

- **Wednesday, September 7, 2016**

  09:30 – 09:50 **Alain Miranville:** *Laudatio*

  09:50 – 10:35 **Roger Temam:** *Modeling some multiphase problems of the humid atmosphere*

  10:40 – 10:50 **Pierluigi Colli:** *Laudatio*

  10:50 – 11:25 **Antonio Segatti:** *Variational models for nematic shells*

  11:30 – 12:10 *break*

  12:10 – 12:45 **Davide Bigoni:** *The geometry of a highly deformed elastic rod, with applications to snakes, catapults, and self-oscillating systems*

  12:45 – 14:00 *lunch*

  20:00 – 23:00 *social dinner:* Casa dell'Aviatore, Viale dell'Università, 20, Roma

- **Thursday, September 8, 2016**

  09:30 – 10:05 **Alexander Mielke:** *On the geometry of reaction and diffusion*

  10:10 – 10:45 **Harald Garcke:** *Cahn–Hilliard–Darcy models for tumour growth with chemotaxis and active transport*

  10:50 – 11:25 **Pavel Krejčí:** *A model for fluid flow in unsaturated deformable porous media*

  11:30 – 12:00 *break*

  12:00 – 12:35 **Elisa Davoli:** *Weighted Energy-Dissipation approach to dynamic perfect plasticity*

  12:40 – 13:55 **Lev Truskinovsky:** *Cell locomotion in one dimension*

  13:20 – 14:30 *lunch*

  14:30 – 15:05 **Andrea Braides:** *Homogenization of spin systems*

  15:10 – 15:45 **Klaus Hackl:** *Modeling of microstructures in a Cosserat continuum using relaxed energies*

  15:50 – 16:20 *break*

  16:20 – 16:55 **Ben Schweizer:** *Periodic and stochastic homogenization of plasticity equations*

- **Friday, September 9, 2016**

  09:30 – 10:05 **Jürgen Sprekels:** *On a nonstandard Cahn–Hilliard system with dynamic boundary conditions*

  10:10 – 10:45 **Marshall Slemrod:** *Fluids, elasticity, geometry, and wild solutions*

  10:50 – 11:20 *break*

  11:20 – 11:55 **Martin Kružík:** *Orientation-preserving homeomorphisms in nonlinear elasticity*

  12:00 – 12:35 **Alain Miranville:** *Cahn-Hilliard inpainting*

Abstracts

The geometry of a highly deformed elastic rod, with applications to snakes, catapults, and self-oscillating systems
Davide Bigoni (Trento)

How do forms emerge and develop during loading of an elastic structure? This question will be addressed within the framework of the nonlinear theory of elastic rods, which is suitable for describing bifurcation and instabilities of a number of interesting structures. Various problems will be addressed, including a model for serpentine motion, a model for an elastic catapult, and self-oscillating systems, which reach a limit-cycle oscillation when subject to a steady input of energy.

Homogenization of spin systems
Andrea Braides (Roma 2)

We consider surface energies arising from spin systems. Those arise in problems in Statistical Physics, but can also be regarded as an approximation of more general (e.g. Lennard-Jones) interactions close to discontinuity surfaces, such as cracks. We address the problem of the description of the macroscopic behaviour of spin systems with inhomogeneous interactions. In particular 1) we completely describe the effective surface tension of periodic mixtures of two types of ferromagnetic interactions. This problem is linked to optimal design of networks and their metric properties (joint work with L.Kreutz); 2) we study conditions ensuring the possibility of describing systems of mixtures of ferromagnetic and antiferromagnetic interactions via an effective surface tension. Examples of frustrated systems are given when this can or cannot be done (joint work with M.Cicalese); 3) we study the case when antiferromagnetic interactions appear with low (but finite) probability, and show that generically such system behave as ferromagnetic systems (joint work with A.Causin, A.Piatnitski, and M.Solci).

Crystalline mean curvature flow
Antonin Chambolle (Paris)

We present a recent result on existence and uniqueness for a class of simple crystalline curvature flows. This is based on a notion of solution which is equivalent to viscosity solutions in smooth cases.

Solutions for a model related to solidsolid phase transitions in shape memory alloys
Pierluigi Colli (Pavia)

A model for solid-solid phase transition in shape memory alloys has been proposed by Fabrizio and coworkers. The physical quantities that are involved are the absolute temperature, the order parameter, and the scalar displacement along with the associated stress vector. In the corresponding PDE system, two parabolic equations and a second order hyperbolic equation are coupled. Some technical difficulties have to be tackled when studying the system; in particular, the equations contain nonlinear coupling terms, higher order dissipative contributions, possibly multivalued operators. The present talk provides the outline of an existence result obtained in collaboration with Elena Bonetti, Mauro Fabrizio and Gianni Gilardi.

Weighted Energy-Dissipation approach to dynamic perfect plasticity
Elisa Davoli (Vienna)

We present a proof of existence of solutions to the equations of dynamic linearized perfect plasticity, based on a global variational formulation of the problem by means of the weighted energy-dissipation approach. This is a joint work with Ulisse Stefanelli.

Cahn–Hilliard–Darcy models for tumour growth with chemotaxis and active transport
Harald Garcke (Regensburg)

In recent years several phase field type models for tumour growth have been introduced. We propose a new model which is derived with the help of basic thermodynamical principles and leads to a Cahn–Hilliard–Darcy system. The model takes nutrient diffusion, chemotaxis, apoptosis, proliferation, adhesion and as a new aspect active transport into account. An additional new feature of the model is that it is based on a volume averaged velocity which leads to particular simple equations for the flow velocity and the pressure.
The overall system is given as

$$
\begin{aligned}
\operatorname{div} \mathbf{v} &= \alpha\Gamma\,, \\
\mathbf{v} &= -K(\nabla p - \mu\nabla\varphi - \chi\sigma\nabla\varphi)\,, \\
\partial_t\varphi + \operatorname{div}(\varphi\mathbf{v}) &= m\Delta\mu + \overline{\rho}_s\Gamma\,, \\
\mu &= \frac{\beta}{\varepsilon}W'(\varphi) - \beta\varepsilon\Delta\varphi - \chi\sigma\,, \\
\partial_t\sigma + \operatorname{div}(\sigma\mathbf{v}) &= \operatorname{div}(n(\chi_\sigma\nabla\sigma - \chi\nabla\varphi)) - \mathcal{C}\sigma h(\varphi)\,, \\
\Gamma &= (\mathcal{P}\sigma - \mathcal{A})h(\varphi)\,.
\end{aligned}
$$

Here, $\mathbf{v}$ is the velocity, p the pressure, φ is related to the tumour density, μ is the chemical potential and σ is the nutrient density. Furthermore α, K, $\overline{\rho}_s$, m, β, ε, n, $\mathcal{C}$, $\mathcal{P}$, $\mathcal{A}$ are positive constants, W is a double well potential and $\alpha \geq 0$. The phase field φ takes the value $+1$ in the tumour region and -1 in the region occupied by healthy cells. The function h is an interpolation with $h(1) = 1$ and $h(-1) = 0$.
In the talk I will derive the model, identify the sharp interface limit in the limit $\varepsilon \to 0$, state well-posedness results and discuss possible generalizations to multi-phase situations. Qualitative and quantitative features of the model will be illustrated with the help of numerical simulations.

Geometric stability for one-dimensional variational problems
Alain Goriely (Oxford)

Given a functional for a one-dimensional physical system, a classic problem is to minimize it by finding stationary solutions and then checking the positive definiteness of the second variation. Establishing the positive definiteness is, in general, analytically untractable. However, in this talk, I will show that a global geometric analysis of the phase-plane trajectories associated with the stationary solutions leads to simple generic conditions for minimality. These results provide a straightforward and direct proof of positive definiteness, or lack thereof, in many important cases. In particular, when applied to mechanical systems, the stability or instability of entire classes of solutions can be obtained effortlessly from their geometry in phase-plane, as illustrated on a problem of a mass hanging from an elastic rod with intrinsic curvature. This work is in collaboration with Thomas Lessinnes.

A model for fluid flow in unsaturated deformable porous media
Pavel Krejčí (Prague)

Thermomechanical interactions between liquid, gas, and deformable porous solid matrix are described in terms of balance laws for conservation of mass, momentum, and energy. In the mass balance, hysteresis between the wetting and dewetting regimes is taken into account in the form of a Preisach operator in agreement with the work done by J. P. O'Kane and D. Flynn. The full system of balance equations is degenerate because of the a priori saturation limit. We also discuss the possibility of considering phase transition phenomena when the liquid freezes and melts in the porous body. Results of recent joint work with B. Albers and E. Rocca will be presented here.

Orientation-preserving homeomorphisms in nonlinear elasticity
Martin Kružík (Prague)

We will review existing approaches to orientation preservation and injectivity of deformations in nonlinear elasticity. Then we find necessary and sufficient conditions for weak* lower semicontinuity of elastic energies defined on bi-Lipschitz orientation-preserving maps in the plane. As an application, we show the existence of a minimizer for an integral functional with nonpolyconvex energy density among bi-Lipschitz homeomorphisms.

On the geometry of reaction and diffusion
Alexander Mielke (Berlin)

We discuss joint work with Matthias Liero and Giuseppe Savaré which concerns the Hellinger-Kantorovich distance on measures over a bounded convex domain $\Omega \subset \mathbf{R}^d$. This distance can be seen as an inf-convolution of the Wasserstein distance and the Helligner-Kakutani distance. We show that based on the new distance we can generalize the theory of Jordan-Kinderlehrer-Otto for the Fokker-Planck equation and nonlinear diffusion equations to reaction-diffusion equations. In particular, we highlight conditions that lead to contractive semiflows with respect to the new distance.

Cahn-Hilliard inpainting
Alain Miranville (Poitiers)

Our aim in this talk is to discuss variants of the Cahn-Hilliard equation in view of applications to image inpainting. We will present theoretical results as well as numerical simulations.

Instabilities and oscillations in coagulation equations
Barbara Niethammer (Bonn)

Smoluchowski's classical coagulation model can describe mass aggregation phenomena in a large variety of applications, such as aerosol physics, polymerization, population dynamics, or astrophysics. The equation is a nonlinear integral equation for the number density of clusters, involving a rate kernel to account for the microscopic details of the specific aggregation process.

Of particular relevance is to understand whether the long-time behaviour of solutions is universal. In this talk I will in particular discuss the case of diagonally dominant kernels of homogeneity one. Here formal arguments lead to the conjecture that for large times the coagulation equation can be seen as a regularization of the Burgers equation. In contrast to diffusive regularization, however, we obtain phenomena such as instability of the constant solution or oscillatory traveling waves.

What does rank-one convexity have to do with viscosity solutions?
Pablo Pedregal (Castilla La Mancha)

We will try to relate the two concepts in the title by means of the celebrated Hilbert's theorem for non-negative polynomials, in the case of 2×2-matrices, that translates into the condition of a related function to be a viscosity sub solution of a certain PDE. In practice, we will come up with a test for rank-one convexity for the 2×2 case.

Balanced Viscosity solutions for multi-rate systems
Riccarda Rossi (Brescia)

Several mechanical systems are modeled by the static momentum balance for the displacement u coupled with a rate-independent flow rule for some internal variable z. We consider a class of abstract systems with this structure, and regularize both the static equation and the rate-independent flow rule by adding viscous dissipation terms with coefficients ε^α and ε, where $0 < \varepsilon \ll 1$ and $\alpha > 0$ is a
fixed parameter. Therefore for $\alpha \neq 1$ the variables u and z have different relaxation rates. We address the vanishing-viscosity analysis as $\varepsilon \downarrow 0$ of the viscous system. We prove that, up to a subsequence, (reparameterized) viscous solutions converge to a parameterized curve, yielding a Balanced Viscosity solution to the original rate-independent system, and providing an accurate description of the system behavior at jumps. We also give a reformulation of the notion of Balanced Viscosity solution in terms of a system of subdifferential inclusions, showing that the viscosity in u and the one in z are involved in the jump dynamics in different ways, according to whether $\alpha > 1$, $\alpha = 1$ and $\alpha \in (0, 1)$. Joint work with A. Mielke (Berlin) and G. Savaré (Pavia).

Non-linear maximum entropy principle for a polyatomic gas subject to the dynamic pressure
Tommaso Ruggeri (Bologna)

We establish Extended Thermodynamics (ET) of rarefied polyatomic gases with six independent fields, i.e., the mass density, the velocity, the temperature and the dynamic pressure, without adopting the near-equilibrium approximation. The closure is accomplished by the Maximum Entropy Principle (MEP) adopting a distribution function that takes into account the internal degrees of freedom of a molecule. The distribution function is not necessarily near equilibrium. To my knowledge, this is the first example of molecular extended thermodynamics with a non-linear closure. The integrability condition of the moments requires that the dynamical pressure should be bounded from below and from above. The model obtained is the simplest example of non-linear dissipative fluid after the ideal case of Euler. The system is symmetric hyperbolic with the convex entropy density and the K-condition is satisfied. Therefore, in contrast with the Euler case, there exist global smooth solutions provided that the initial data are sufficiently smooth.

Variational models for nematic shells
Antonio Segatti (Pavia)

In this talk I will discuss the main features of a variational model for nematic shells. After introducing the model, I will present some recent results dealing with qualitative properties of the solutions. Among all, I will concentrate on the relations between the topology of the shell and the functional framework of the problem. If time permits, I will discuss some current research dealing with the emergence of defects. The talk is based on jont works with G. Canevari (Oxford), M. Veneroni (Pavia) and M. Snarski (Brown).

Fluids, elasticity, geometry, and wild solutions
Marshall Slemrod (Madison)

This talk gives a short review of recent work of A. Acharya (CMU), G.-Q. Chen (Oxford), M. Slemrod (Univ. of Wisconsin-Madison), D. Wang (Pittsburgh) on the direct connection between the 2d Euler equations of incompressible fluid flow and motion of a 2d surface in 3-dimensional Euclidean space. In particular we note how this link provides a direct application of the Nash-Kuiper non-smooth isometric embedding result which in turn provides information on ill-posedness for the initial value problem for weak solutions of the Euler equations as well as neo-Hookian elasticity. Furthermore it suggests that the Nash-Kuiper solutions may represent fluid turbulence. The work was strongly motivated by the sequence of papers of C. De Lellis and L. Szekelyhidi, Jr.

On a nonstandard Cahn–Hilliard system
with dynamic boundary conditions
Jürgen Sprekels (WIAS Berlin)

We study a nonstandard Cahn–Hilliard system that was originally introduced by P. Podio-Guidugli as a model for phase separation on an atomic lattice. The model leads to a strongly coupled system of PDEs for the two unknowns of the model, the chemical potential and the order parameter (which is the density of one of the phases). The couplings between the unknowns are nonlinear even in the time derivatives. While numerous contributions to the analysis of this system exist for the case of zero Neumann boundary conditions, we consider here for the first time dynamical boundary conditions involving the Laplace–Beltrami operator, which model the occurrence of an additional nonconserving diffusive phase transition on the boundary. The corresponding bulk and surface contributions to the free energy of the system are allowed to contain nonlinearities of logarithmic or of double obstacle type. For the resulting initial-boundary value problem, we prove well-posedness results. The existence proof combines retarded argument approximations, Moser type estimates, and monotonicity and compactness arguments. This is a joint work with P. Colli and G. Gilardi from the University of Pavia.

Periodic and stochastic homogenization of plasticity equations
Ben Schweizer (Dortmund)

he plastic deformation of a piece of metal is commonly described with small-strain plasticity equations of von-Mises type. We are interested in the case that the coefficients of these equations are not homogeneous, but that they show oscillations on a spatial scale $\varepsilon > 0$, either periodic in x or stochastic. In a model with kinematic hardening, we derive the effective system which describes the behavior of solutions in the limit $\varepsilon \to 0$. In contrast to many standard homogenization problems, where the limit system has the same structure as the original system, we observe here an interesting difference: The plasticity system includes hysteresis and hence contains some memory effect. In the homogenization process, the coupled elementary processes are averaged and lead to a much more complex memory term. We use either two-scale convergence and or the needle-problem approach to homogenization. The results are obtained jointly with M. Veneroni and with M. Heida.

Modeling some multiphase problems of the humid atmosphere
Roger Temam (Indiana)

In this lecture, I will described some multiphase problems connected to the humid atmosphere. I will present the modeling of the problems, and the study of the existence, uniqueness and regularity of solutions to the problems.

Rate-independent evolution of sets
Marita Thomas (WIAS Berlin)

This contribution deals with the rate-independent evolution of a set Z of finite perimeter. Its evolution is governed by the interplay of a time-dependent forcing term with the perimeter-term and with a rate-independent dissipation metric related to the volume. We give a notion of solution in terms of a stability condition, discuss fine properties of solutions, and study the validity of an energy-dissipation balance. From this abstract set-model we draw links to delamination problems, where the set Z is related to the crack-set. This is joint work with R. Rossi (Brescia) and U. Stefanelli (Vienna).

Kinetic theory of wealth distribution
Giuseppe Toscani (Pavia)

In the last decade, kinetic theory has proved a very effective tool in solving problems in social sciences and economics [1]. In particular, the distribution of wealth in a multi- agent society has been investigated by resorting to classical methods of kinetic theory of rarefied gases. In analogy with the Boltzmann equation, the change of wealth in these models is due to microscopic binary trades among agents. In this lecture, we present and discuss some recent results in the field. Among others, we consider the possible role and in uence of knowledge in the evolution of wealth in a system of agents which interact through binary trades [2]. The trades, which include both saving propensity and the risks of the market, are here modified in the risk and saving parameters, which now are assumed to depend on the personal degree of knowledge. The numerical simulations show that the presence of knowledge has the potential to produce a class of wealthy agents and to account for a larger proportion of wealth inequality.

References

[1] L. Pareschi, G. Toscani, Interacting Multiagent Systems: Kinetic Equations and Monte Carlo Methods, Oxford University Press, Oxford (2014)
[2] L. Pareschi, G. Toscani, Wealth distribution and collective knowledge. A Boltzmann approach, Phil. Trans. R. Soc. A 372, 20130396, 6 October (2014)

Cell locomotion in one dimension
Lev Truskinovsky

We discuss a class of mathematical models of lamellipodial cell motility on a substrate (crawling) that are based on a projection of a complex intra-cellular dynamics into one dimension.

Anisotropic spatial variations in liquid crystal models
Arghir Zarnescu (Sussex)

The specific mathematical feature of liquid crystal models is that one works with functions taking values into certain manifolds. The physical and material symmetries then impose restrictions on the types of spatial variations allowed in the energy functionals. Apart from the usual Dirichlet energy there are certain combinations of first order derivatives, that generate in the corresponding Euler-Lagrange equations matrix-valued elliptic operators that are far from being diagonal.
The equations then exhibit a number of specific and intriguing features, some of which will be explored during the talk.
