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Bloch waves in a triangular lattice with tilted resonators: applications to focussing

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Abstract – **We consider a vibrating triangular mass-truss lattice whose unit cell contains a rigid resonator. The resonators are linked by trusses to the triangular lattice nodal points. We assume that the resonator is tilted , *i.e.* it is rigidly rotated with respect to the triangular lattice's unit cell by an angle ϑ_0. This geometric parameter controls a resonant mode in the spectrum for elastic Bloch waves and affects the dispersive properties of the lattice. We provide physical interpretations of these phenomena and discuss the dynamic implications on elastic Bloch waves. In addition, we describe a structured interface containing tilted resonators which exhibits focussing by negative refraction, as in a "flat elastic lens".**

I. Introduction

In the last two decades, significant progress has been made in the control of elastic waves at the micro-scale. This advance is primarily due to recent improvements in micro- and nano-fabrication as well as the novel theoretical developments, such as focussing through negative refraction. The use of negative refraction for focussing was studied theoretically by Pendry [1]. Later, Luo *et al.* [2] exploited the dynamic anisotropy of a photonic crystal to achieve all-angle negative refraction and focussing without a negative effective index of refraction. The same idea has been exploited in the context of phononic crystals for both elastic [3, 4] and water [5] waves as well as waves in lattice systems [6, 7].

Brun *et al.* [8] introduced an analytical model for geometrically chiral elastic media, which takes into account internal rotations induced by gyroscopes. These structured media act as polarisers of elastic waves. Further studies of chiral structures have lead to exciting and novel results [9, 10, 11]. Hexagonal chiral lattices made of massive rods were studied by Spadoni *et al.* [12, 13] and by Liu *et al.* [14, 15]. Their numerical results are in the framework of continuum elasticity and highlight the effect of geometric chirality on the dynamic anisotropy of elastic in-plane waves and on the auxetic behaviour of the structure under static loads. In particular, it has been demonstrated in [12] and [14] that the geometric chirality enabled the breakage of the reflection symmetry in the high-order dispersion surfaces. More recently Rosi and Auffray [16] rediscovered the same structure in the framework of the strain-gradient continuum elasticity. The observation of dynamic chirality in structured elastic materials has been reported by Süsstrunk and Huber [17]. In particular it has been demonstrated that left and right propagating collective edge waves are not equivalent, *i.e.* the system is dynamically chiral. The seminal experiment reported in [17] led to the newly established research field of "topological mechanics" [18].

An elastic lattice, with its inertia concentrated at the nodal points, is a highly attractive object for the study of Bloch waves, as the dispersion equations can be expressed as polynomials with respect to a spectral parameter. The paper by Martinsson and Movchan [19] presented a unified analytical approach for the study of Bloch waves in multi-scale truss and frame structures, which included problems of the design of stop bands around pre-defined frequencies. The asymptotic analysis of eigenvalue problems for degenerate and non-degenerate multi-structures was systematically presented in the monograph by Kozlov *et al.* [20], where rotational modes were studied in the context of the asymptotic analysis of the eigenvalues and corresponding eigenfunctions of multi-structures consisting of components of different limit dimensions.

II. Results

In this section we highlight the main results from [21], where we studied the Bloch problem for the linear in-plane motion of a triangular lattice whose unit cell contains a tilted resonator. The Bloch frequency dispersion

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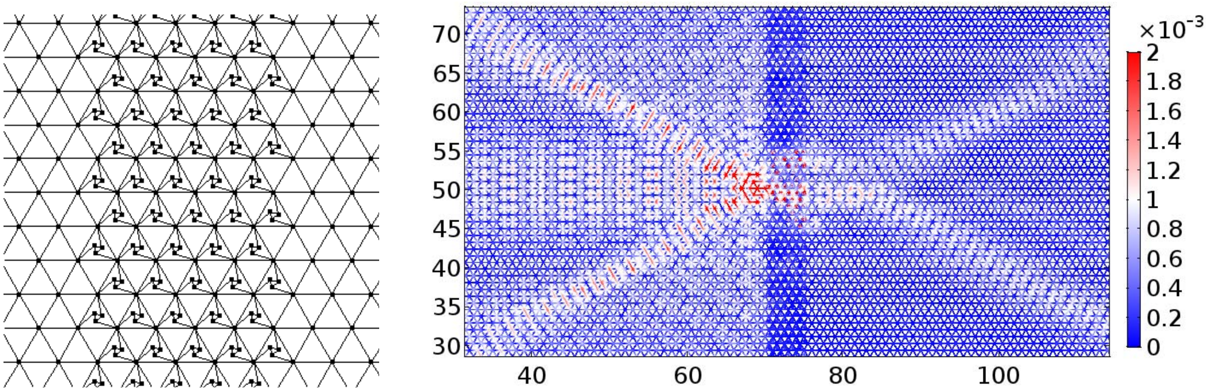

Fig. 1: A structured interface (left panel) made of tilted resonators in a triangular elastic lattice exhibits focussing by negative refraction (right panel). The picture shows the modulus of the displacement field generated by a harmonic point load, horizontally polarised and of frequency $\nu = \pi$ rad/s. The physical parameters are $c_{\rm TL} = 9$ N/m, $c_{\ell o} = 1.36 c_\ell$, $m_{\rm TL} = 4$ Kg, $m = 1$ Kg, $m = 3$ Kg and $\vartheta_0 = 1.36$ rad/s. The lattice spacing is $L = 1$ m and the resonators are equilateral triangles of side $\ell = 0.21$ m.

surfaces for a triangular lattice where the nodal points are linked to a point-wise resonator, comprise two acoustic branches and two optical branches, separated by a complete band gap [19]. Instead of a point-wise mass, we recently studied [21] an extended triangular rigid body with concentrated masses at the vertices. The vertices are connected to the triangular lattice nodal points by massless trusses. The resonators are rotated with respect to the unit cell of an angle ϑ_0. The rotational degree of freedom of the resonators about their centres of mass gives rise to a novel Bloch mode. In particular, the frequency of the Bloch mode and its effective properties, such as group velocity and effective mass tensor, can be controlled by the tilting angle.

The tunability of the novel Bloch mode allows us to design and implement a structured interface able to focus elastic waves (see Fig. 1). Consider a triangular lattice where the mass of the nodal points is $m_{\rm TL}$ and the stiffness of the links is $c_{\rm TL}$. The structured interface (left panel in Fig. 1) is embedded in the ambient triangular lattice by introducing a finite layer of tilted resonators, one per unit cell. The total mass M of each resonator is equally distributed between the vertices. Within the interface the mass has been redistributed in such a way that $m_{\rm TL} = M + m$, where m is the mass of the triangular lattice nodal points at the interface. On the left half plane, an harmonic point load of amplitude $F = 0.1\ N$, linearly polarised in the horizontal direction, acts on a triangular lattice nodal point. The angular frequency of the excitation is $\nu = \pi$ rad/s. The directionally localised displacement field generated by the point force is consistent with the preferential directions of the lattice at the frequency of interest. The cross-like pattern experiences negative refraction at the boundaries of the interface and emerges on the right side as a real image.

III. Conclusion

In Ref. [21] we have introduced and studied a new class of chiral lattice system which exhibits a range of interesting dynamic properties, including dynamic anisotropy, negative refraction, filtering and focusing of elastic waves. The chiral lattice is created by the introduction of a periodic array of triangular resonators embedded within an infinite uniform triangular lattice; the chirality arises as a result of *tilting* the resonator and thus breaking the mirror symmetry of the ambient lattice.In particular, the resonators give rise to a novel rotationally dominant mode, which we refer to as the “chiral branch”. We use this chiral branch to design and implement a flat lens for mechanical waves in a triangular lattice.

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