CONFIGURATIONAL FORCES IN ELASTIC STRUCTURES: INJECTION & DRIPPING OF ELASTIC RODS

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Abstract: *Simple elastic systems are presented in which configurational forces develop as the response of the mechanical system to the possibility of changing its configuration through a release of elastic energy. The discovery of these Eshelby-like forces acting on structures has opened new possibilities in the design of measuring devices and actuators and has provided a new insight in the problems of 'injection' and self-encapsulation of elastic rods.*

1. Introduction

Configurational forces, as introduced by J.D. Eshelby in famous articles [1-2], are the cornerstone of a well-developed theory, in which important contributions have been given, among others, by Kienzler and Herrmann [3-4], Gurtin [5], Maugin [6], Steinmann [7].

Recently, Bigoni, Bosi, Dal Corso, and Misseroni have shown that configurational forces can be easily generated in elastic structures [8]. These forces have been shown to deeply influence structural stability [9], and have inspired the design of a new type of scale, in which both equilibrium (as in a rigid arm balance) and deformation (as in a spring balance) determine the solution of a highly nonlinear system that can be calculated and realized to measure weight [10]. Furthermore, configurational forces can also be induced by torsion, a concept which has inspired the idea of a new type of torsional actuator, nicknamed 'torsional gun' [11].

It is well-known that the Euler's differential equation of the elastica governs an oscillating pendulum, a buckling rod, and a pendant drop. This observation has inspired the two problems that will be addressed, namely, i.) the 'injection' of an elastic rod through a sliding sleeve against a fixed obstacle, Fig. 1, and ii.) the 'dripping' of an elastic rod or its 'self-encapsulation'. These problems, where equilibrium is governed by configurational mechanics, are analytically solved and experimentally verified.

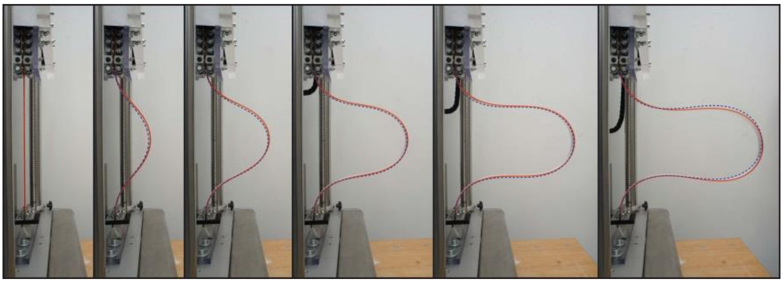

Fig. 1 A sequence of photos during the injection of an elastic rod, where a configurational force is developed at the sliding sleeve (upper constraint). Theoretical elastica are reported as dashed lines.

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