Folding and Stress Channelling in Cosserat Continua

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It is well known that in classical Cauchy materials with extreme orthotropic properties the stress does not diffuse in the medium and the solution becomes highly localized and strongly directional. In fact, in the limit when the stiffness ratio between different material directions tends to zero, the equations governing equilibrium reach the elliptic boundary and the stress percolates through *null*-thickness deformation bands. This phenomenon is called stress channelling and occurs in highly orthotropic fibre-reinforced materials where disturbances can propagate along fibers without attenuation [1]. Stress channeling effects have been observed also experimentally in masonry models by Bigoni and Noselli [2,3]. In general, materials with extreme anisotropy can be exploited in different technologies, for instance, in mechanical wave guiding, stress wave shielding, and invisibility cloaking.

The purpose of the present work is to analyze localization phenomena in elastic materials with extreme orthotropic properties in the framework of couple-stress elasticity. This theory introduces characteristic material lengths in order to describe the scale effects that emerge from the underlying microstructure and has proved to be very effective for modeling complex materials. In this context, the notions of ellipticity (E), strong ellipticity (SE), and the conditions for wave propagation (WP) are established. The Green's functions for a concentrated force and a concentrated moment are obtained analytically for an orthotropic couple-stress material, where the acoustic tensor is introduced ruling bulk plane-wave propagation. The Green's functions allow the investigation of the material behavior near the (E) boundary, in the spirit of Bigoni and Capuani [4,5]. It is observed that when (E) is lost *folding* occurs in the Cosserat material, a phenomenon that has no counterpart in the classical context. On the other hand, when the (WP) condition fails, the localization percolates is a band of *finite* thickness. The order of magnitude of this band is strongly related to the material microstructure.

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