ESHELBY-LIKE FORCES AND BUCKLING OF SIMPLE ELASTIC STRUCTURES

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Abstract. *Eshelbian force are known to be connected with the propagation of defects in solids. The purpose of this presentation is to show how these forces can be theoretically and experimentally found in elastic structures and to quantfy their effects on buckling and instability.*

Keywords: Buckling, Eshelbian forces, material forces, Elastica.

1. INTRODUCTION

The Eshelbian force is the main concept of a celebrated theoretical framework associated with the motion of dislocations, phase transforming boundaries, and defects in solids. In a similar vein, in an elastic structure where a (smooth and bilateral) constraint can move and release energy, a force driving the configuration change is generated, which therefore is called by analogy ‘Eshelby-like’ or ‘configurational’. This force (generated by a specific movable constraint) can be derived via variational calculus or through an asymptotic approach. Its action on the elastic structure is counterintuitive, but is fully substantiated and experimentally measured on a model structure that has been designed, realized and tested.

In another model structure, an elastic rod subject to a dead compressive load at the free end penetrates into a sliding sleeve, ending with a linear elastic spring. Bifurcation and stability analysis of this simple elastic system shows a variety of unexpected behaviours: (i.) an increase of buckling load at decreasing of elastic stiffness; (ii.) a finite number of buckling loads for a system with infinite degrees of freedom (leading to a non-standard Sturm-Liouville problem); (iii.) more than one bifurcation loads associated to each bifurcation mode; (iv.) restabilization of the straight configuration after the second bifurcation load associated to the first instability mode; (v.) the presence of an Eshelby-like (or configurational) force, deeply influencing stability.

The findings reported in the presentation open a new perspective in the mechanics of deformable mechanisms, with possible broad applications, even at the nanoscale.

2. THE ANALYZED STRUCTURES

The elastic structure used to reveal an Eshelby-like force is shown in Fig. 1. Its peculiarity is the presence of a sliding sleeve leaving the structure with the possibility of moving and releasing elastic energy.

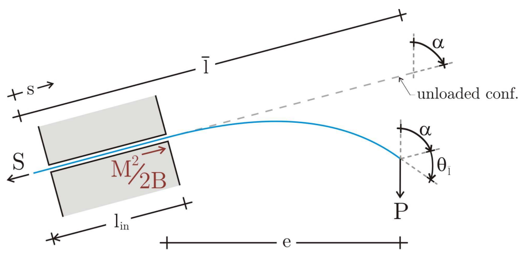

Fig. 1 Structural scheme of the elastic system employed to disclose a Eshelby-like force. The elastic rod of total length l is subject to a dead vertical load P on its right end, is constrained with a sliding sleeve inclined at an angle α (with respect to the vertical) and has a axial dead force S applied at its left end. The presence of the Eshelby-like force $M^2/(2B)$ influences the force S at equilibrium, which results different from P cos α.

An Eshelby-like force is also present in the structure shown in Fig. 2 and here it strongly influences stability and the postcritical behavior. The structure is an elastic blade slipping without friction inside a sliding sleeve and restrained with an elastic spring.

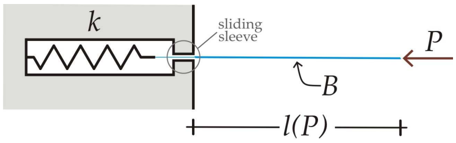

Fig. 2 The penetrating blade is an elastic planar rod whose free length l is a function of the amount of the applied axial load P. The blade with constant bending stiffness B has a free end subject to the dead load P, while at the other edge the blade slides into a frictionless sleeve and is restrained by an axial linear spring of stiffness k.

The two structures are analyzed in detail and reveal a number of ‘unexpected’ features related to the presence of the configurational force.

3. ACKNOWLEDGEMENTS

D. B. gratefully acknowledges ERC Advanced Grant “Instabilities” 2013, number 340561.

4. REFERENCES

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Bigoni, D., Bosi, F., Dal Corso, F., and Misseroni, D. 2014. Instability of a penetrating blade. J. Mech. Phys. Solids, in press.

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