Self-shaping & energy releases of highly deformed elastic bodies, with applications to material instabilities and Euler's elastica

Davide Bigoni
DICAM – University of Trento
Via Mesiano 77 – 38123 Trento
email: davide.bigoni@unitn.it
website: http://www.ing.unitn.it/~bigoni/

Emerging morphologies in elasticity will be investigated for extreme materials, namely, solids designed to work near a material instability, where they display stress channeling and strain localization.

The perturbative approach to shear bands will be introduced with reference to nonlinear elasticity, deformed incrementally [1]. It will be shown how this approach is tailored to reveal features of shear band propagation, including stress concentrations and dynamic effects. Finally, extreme couple stress solids will be introduced to explain the formation of folding and faulting, occurring when the material is near the threshold of loss of ellipticity [2, 3, 4], see Fig. 1.

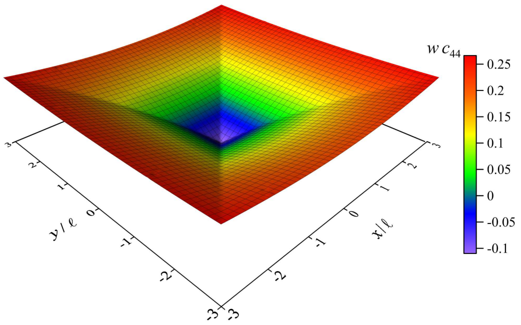

Fig. 1 - Cross folding of a constrained Cosserat elastic material subject to a concentrated force near the boundary for failure of ellipticity

The problem of an elastic rod deforming in a plane, namely the so-called 'planar elastica', has a long history, rooting to Jacob Bernoulli (1654-1705), Daniel Bernoulli (1700-1782), Leonhard Euler (1707-1783), and Pieter van Musschenbroek (1692-1761), but is still actual and rich of applications, sometimes unexpected. The elastica has attracted a great interest in the past and has involved contributions from first-class scientists, including Kirchhoff, Love, and Born. The research on the elastica marked the initiation of the calculus of variations and promoted the development of the theory of elliptic functions. Nowadays the elastica represents a useful introduction to the theory of nonlinear bifurcation and stability, but is also an important tool in the field of soft robotics and in the design of compliant mechanisms.

Using the elastica theory, configurational or 'Eshelby-like' forces will be shown to arise in elastic structures when a change in configuration is possible, with a related release of energy. This concept has been developed theoretically and experimentally in a series of recent works involving: a clamped elastic rod forced to slip inside a sliding sleeve [5], the development of the so-called 'elastica arm scale' [6], the development of an elastica in the shape of a drop [7, see Fig. 2], an example of torsional locomotion [8] and serpentine motion within a smooth channel.

Fig. 2 The self-encapsulation or the 'dripping' of an elastic rod compared with the formation of a water drop

Experiments are presented on an elastic rod, straight in its unloaded configuration, constrained within a rigid and frictionless curved channel. A proof-of-concept model for the channel will be shown, with the shape of an Euler spiral, in which friction is reduced to a negligible amount by using roller bearings. Finally an example of energy release through a snap instability will be shown on a real model to be related to the mechanical behaviour of a soft robot arm and also to the working principle of the so-called 'da Vinci catapult' [9].

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Proposed syllabus

1. Solid mechanics at finite strain. Nonlinear elasticity. Incremental equations. Positive definiteness of the constitutive operator, strong ellipticity, ellipticity and wave propagation.
2. Incremental perturbations to a deformed solid: derivation and use of Green's functions. Material instabilities: shear bands and flutter instability in a continuum.
3. The Euler's elastica: nonlinear equations, the Sturm-Liouville problem, and solution via elliptic functions.
4. Playing with the elastica: instability of structures and the Leonardo da Vinci catapult.
5. Configurational or 'Eshelby' forces in elastic structures: elementary examples and variational treatment. Application of a concept: the elastica arm scale, the torsional gun, the snaking, and instability islands.
6. Self-oscillating structures: how a steady energy input can be turned into an oscillation with designed frequency. Flutter instability in a structure and the Tacoma bridge.
7. Conclusions: theory and experiments love each other in mechanics.

References

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