Elastic rods, snakes, and catapults

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Abstract

Configurational or ‘Eshelby-like’ forces arise in elastic structures when a change in configuration is possible with a related release of energy. This concept has been developed theoretically and experimentally in a series of recent works involving: a clamped elastic rod [1], a problem of instability [2], the development of the so-called ‘elastica arm scale’ [3], the ‘injection’ of an elastic rod [4], the development of an elastica in the shape of a drop [5], and an example of torsional locomotion [6]. In the presentation serpentine motion within a smooth channel is theoretically investigated and fully validated by systematic experiments. The experiments are performed on an elastic rod, straight in its unloaded configuration, constrained within a rigid and frictionless curved channel. A proof-of-concept model for a channel is designed and realized, with the shape of an Euler spiral, in which friction is reduced to a negligible amount by using roller bearings.

Gray was the pioneer in the study of snake propulsion and he identified four mechanisms for snake locomotion: serpentine motion, concertina movement, side-winding, and rectilinear propulsion, see [7,8,9]. The principal mechanical concept underlying serpentine locomotion was explained by Gray in terms of a release of elastic energy generated through muscular contraction. His model, based on a chain of rigid pieces connected to each other with rotational springs, explains the counterintuitive phenomenon where locomotion can occur in the absence of friction. This is also in agreement with the experimental observation that serpentine motion ceases when a snake is confined to a straight or circular channel.

The presentation focuses on the theoretical derivation and the experimental proof of serpentine (or undulatory) motion, the first type of the four mechanisms for of snake locomotion. We consider an elastic rod, straight in its undeformed configuration, and constrained within a rigid and frictionless curved channel as shown in Fig. 1.

The propulsive force is generated through a release of elastic energy stored within the elastic rod and is strictly connected to the development of Eshelby-like (or configurational) forces. The propulsive force is derived through different approaches: (a) energetic formulation, (b) integration of the equations of motion, complemented by a ‘micromechanical’ derivation of the channel reactions acting on the elastic rod and providing the Eshelby-like forces.

Finally, a frictionless channel with the shape of a clothoid spiral was designed and realized. The friction between the rigid channel and the elastic rod was reduced by employing modified roller bearings. The experiments, performed on elastic rods with uniform and variable bending stiffness, fully confirm the theoretical predictions.

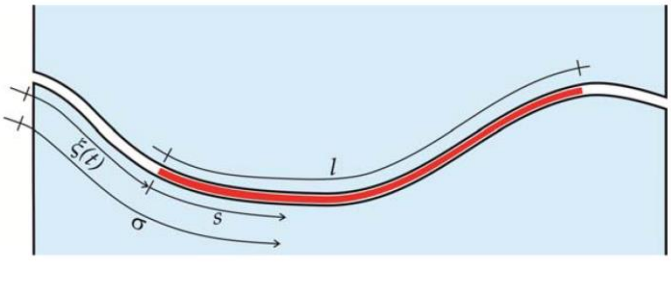

Figure 1. An inextensible elastic rod (rectilinear in its undeformed configuration) of length l within a smooth curvilinear channel. Its configuration is defined by the evolution in time of the curvilinear coordinate $\xi(l)$.

Finally, a model of a so-called ‘elastic catapult’ is considered, in which a clamped elastic rod is assumed to move through a rotation of the clamp. When the free end of the rod is subject to a dead loading greater than that which would cause buckling in the straight configuration a snap-instability occurs, providing a sudden release of elastic energy. The obvious application of the developed theory is soft robot arm.

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