A simplified computational model for the identification of nonlocal parameters of homogenous materials equivalent to Cauchy composites

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Keywords: second displacement gradient elasticity, nonlocal homogenization, periodic materials

The equivalence between a nonlocal elastic homogeneous material and a heterogeneous material with elastic Cauchy phases has been derived through several theoretical approaches, among which: (*i*) the nonlocal (second-order) variational-asymptotic homogenization for periodic material proposed in [1], which provides the analytical expression for the effective nonlocal constitutive tensor. The result depends on the cell microstructure through the perturbation functions, solution of the boundary value problem for the cell subject to period boundary conditions. (*ii*) The analytical asymptotic identification proposed in [2], where quadratic displacement boundary conditions are considered on a single cell containing a small inclusion. The result is given in a closed form expression as linear function of the volume fraction.
While the former, despite of its reliability for every cell properties, is very demanding in terms of computational costs, the latter provides a very simple formula that does not require any specific computation for the higher-order parameters, in addition to that performed for the local constitutive tensor.
We herein propose a simplified second-order homogenization technique based on both the above mentioned approaches with the purpose to provide a reliable and simple tool for the identification of higher-order parameters of effective homogeneous materials. This is done by estimating the perturbation functions through the numerical solution for a multi-cell (cluster of periodic cell) subject to quadratic displacement boundary conditions. A specific quadratic finite element has been implemented in FEAP to access the deformation and the deformation gradient in each gauss point. The overall elastic tensor for the second displacement gradient continuum is determined via an energy matching with the considered multi-cell. Converge of the result by increasing the size of the multi-cell problem is proven.

Acknowledgement: AB and MP would like to thank the European Research Council for the ERC Starting Grant “Multi-field and multi-scale Computational Approach to Design and Durability of PhotoVoltaic Modules” (ERC Grant Agreement n. 306622 – CA2PVM) granted under the European Union’s Seventh Framework Programme (FP/2007-2013). FDC and DB gratefully acknowledge financial support from the ERC Advanced Grant ‘Instabilities and nonlocal multiscale modelling of materials’ FP7-PEOPLE-IDEAS-ERC-2013-AdG (2014-2019).

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