Floquet-Bloch waves in periodic networks of the Rayleigh beams

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We present a novel analysis of a transmission problem for periodic lattices of flexural beams incorporating conventional Euler–Bernoulli beams as well as Rayleigh beams, highlighting the difference in their dynamic response. The two models differ in the presence of the rotational inertia, which is commonly neglected in conventional models of Euler-Bernoulli beam.

Although in the quasi-static regime such structures respond similarly (Fig 1a), as the frequency increases the difference between the Rayleigh and the Euler–Bernoulli beams becomes significant showing interesting features and remarkable dispersion properties (Fig 1b).

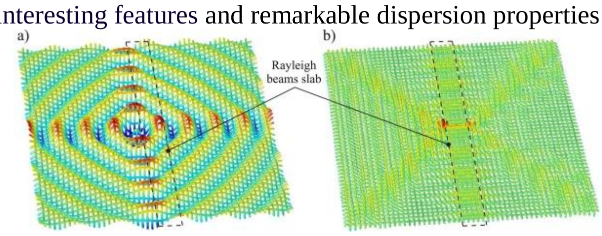

Fig. 1 -Wave propagating in a Euler–Bernoulli beam network and interacting with a structured interface of Rayleigh beams.

We construct the dispersion surfaces for square, rectangular and honeycomb periodic flexural lattices and we give a comparative analysis for both the Rayleigh beams as well as the Euler-Bernoulli beams. Special interest is deserved by the so-called Dirac points representing multiple roots of the dispersion equation, where the dispersion surfaces may become non-smooth. Due to the rotational inertia the frequency of the Dirac point for the lattice consisting of the Rayleigh beams is lower than the frequency of the Dirac point identified for the Euler–Bernoulli beam lattice. It is demonstrated that the rotational inertia is significant for the high-frequency dynamic response, especially in problems involving the Dirac cone steering and analysis of the dispersion degeneracies as well as directional anisotropy and special refraction properties.

From the dispersion surfaces and the isofrequency contours (or slowness contours) we obtain significant information about standing wave regimes, band gaps and dynamic anisotropy. Describing the response of periodic structures of beams in the cases of Euler–Bernoulli beams and the Rayleigh beams, we identify regimes for which a periodic lattice, with interface built of the Rayleigh beams, subjected to an incident wave shows peculiar properties of waveguide, of flat lens, or mirror features.

The results are accompanied by different numerical simulations, where are illustrated forced networks of Rayleigh and Euler–Bernoulli beams with directional localisation, negative refraction, focusing at an interface, and neutrality for propagating plane waves across a structured interface.

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